Transformer SPICE Model

Edward Herbert February 14, 2008

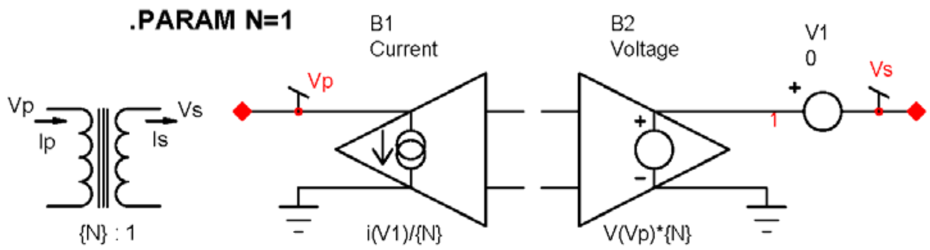
I have been frustrated with currently available SPICE models for transformers, particularly SPICE models that included the core losses and saturation for power when applied to power converter applications. This paper presents new SPICE models that I hope are useful.

The manufacturers' data for core loss parameters, both for real transformer designs and for SPICE models, is woefully inadequate. This paper includes suggestions for more useful data as well as hints to mine more information from present data.

This presentation includes an appendix "Core losses in SPICE models from core manufacturers' data". The reader may wish to look over this section before studying the SPICE models, as the relationships developed in the appendix are the basis for some of the SPICE models.

Contents:

Basic ideal transformer:



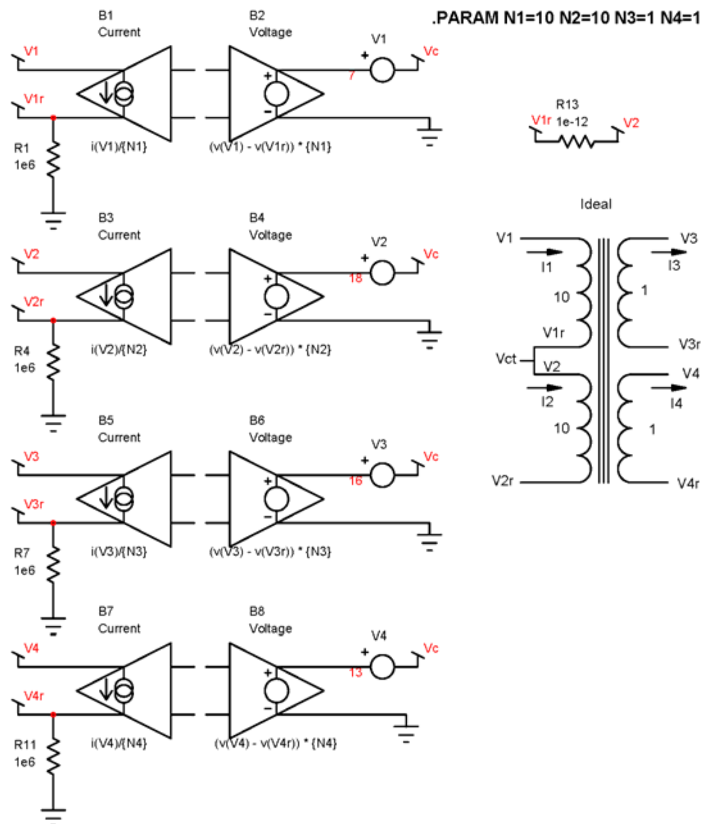
The SPICE model above shows a basic ideal transformer model using a behavioral current source and a behavioral voltage source. The voltage source V1 is used as a current reference for the behavioral current source B1. V1 is set to 0 V so that it has no effect on the circuit. The turns-ratio may be set in the behavioral functions, but I chose to make it a parameter {N}, set using a SPICE parameter statement ".PARAM N=1" so that it is easy to vary without editing the SPICE model itself. The returns are common in this model, but they may be isolated.

This is developed into a family of SPICE models and SPICE model components of increasing complexity (multiple windings, saturation, core loses for high and low frequency, winding losses). The simpler models are useful for many applications, and simplicity is good if it does the job.

Multiple-winding ideal transformers

There are a number of ways to model a multiple-winding transformer in SPICE, but I have chosen to normalize all windings to a one turn linking connection, terminal Vc.

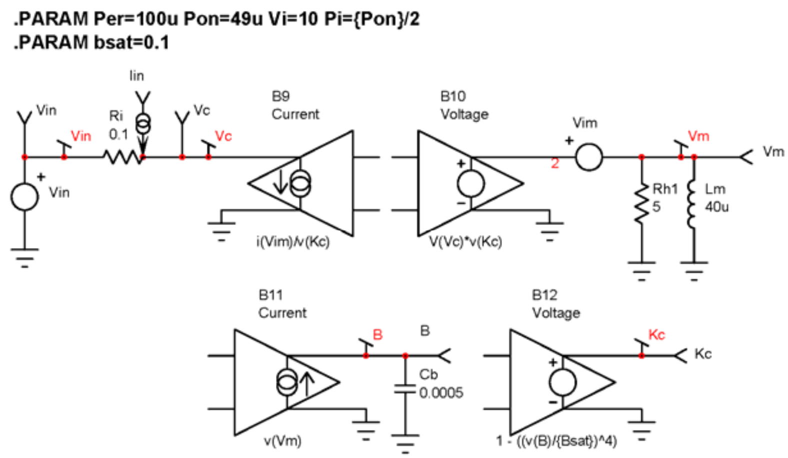
The SPICE model below is for a four winding transformer having a 10 turn push-pull primary winding and two 1 turn secondary windings. In the SPICE model, each section of a multi-tap or split winding is modeled separately, with a separate turns parameter for each ({N1}, {N2}, etc). Each is referred to a single turn "core" winding, connected by the common terminations Vc. No distinction is made whether a winding is a primary or secondary winding in the model, as transformers are reciprocal devices. Any external interconnections are made after the winding sections are defined. The windings are isolated, but in SPICE, having a connection to ground is preferred, so the isolated windings are each connected to ground using very high value resistor, 1 MΩ in the example.



If the transformer being modeled has windings that are connected together to make a tapped winding, it is preferred to connect sections through a low value resistance, so that the node names are not altered, but that is discretionary. In the example, R13 (12 pΩ) connects the nodes V1r and V2 to make the center-tap of the primary winding. Using the example above, it is very easy to make an ideal SPICE model for a transformer having any number of windings. Some care must be taken with nomenclature for "turns". The windings may also be called a 20 turn center-tapped primary with a 2 turn split secondary winding.

Magnetizing inductance, saturation and hysteresis, Simple model:

The SPICE model core functions are coupled to the single turn linking connection Vc through a single turn transformer model. A voltage source is needed to operate the SPICE model, and a square wave ac is preferred.



The resistor Ri limits the current when the core saturates and also is the magnetizing current measurement point for the hysteresis loop display. The voltage source Vim is the current reference for the behavioral current source. Vim is set to 0 V so that it does not affect the circuit.

Caution: Current can be measured in a component. DO NOT use a component as a current measurement point if the measurement is used in the circuit in any way that might feedback to change the current, even minutely. It will cause errors, slow the simulation and may prevent convergence. Use a voltage source set to 0 V.

The inductor Lm models the magnetizing inductance and the resistor Rh1 models the core losses. The core loss will be modeled with a more complex circuit later, but this is a useful starting point and is sufficient for many applications.

The flux B is modeled as the volt-seconds on the inductor Lm, scaled appropriately. The voltage Vm is integrated with respect to time with the behavioral current source B3. The current charges the capacitor Cb to a voltage B. The value of Cb is the scaling factor to convert volt-seconds to flux. Volt-seconds, flux and flux density differ only by scale factors, so any of them may be modeled.

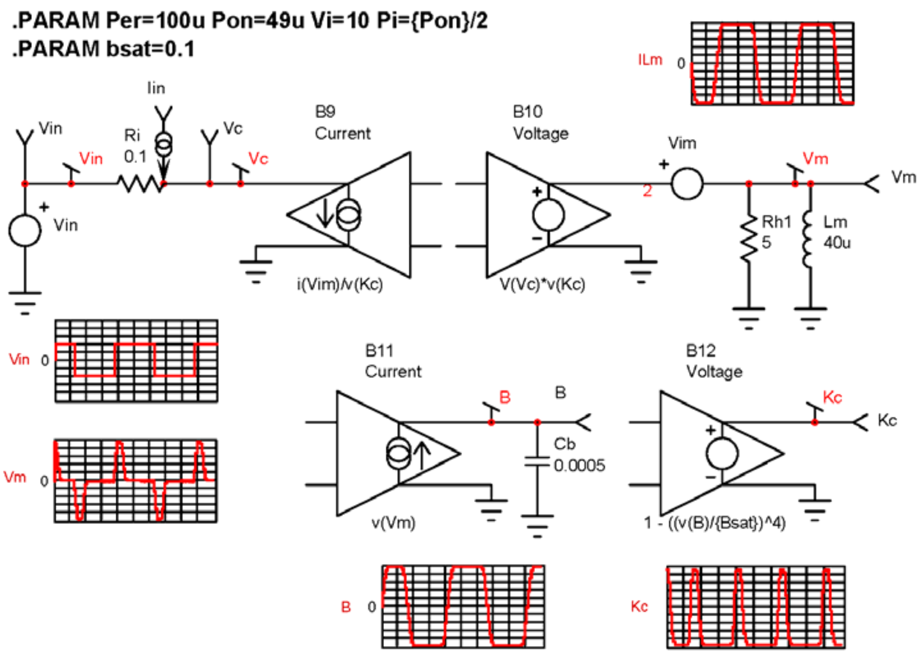
Core saturation is modeled as a coupling factor, Kc. The inductor value and current must remain static in saturation to conserve energy, and the flux B is asymptotic to the saturation flux {bsat}. For the current to remain static, the voltage Vm across the inductor Lm must go to zero, and this is done by reducing the coupling factor Kc to 0 as the flux B goes to {bsat}. There are many functions that can model this behavior, but the following was chosen for its simplicity and versatility:

$$Kc = 1 - \left(\frac{B}{Bsat} \right)^{Exponent}$$

For any exponent > 1 of the expression B/Bsat, Kc goes to 0 asymptotically, and the exponent controls the sharpness of the "knee", a higher exponent making the knee sharper.

To model the hysteresis loop of a magnetic core using conventional units, scale factors to convert the input current to coercive force and to convert volt-seconds to flux density can be used. However, for modeling a transformer, it may be more useful to work with the primary ampere-turns and volt-seconds per turn.

The SPICE model is repeated below, with small graphs that show some of the signals and their timing. The parameter statements used are copied and pasted to the upper left corner. The small graphs are made using the SPICE probe function, and they carry over to a CAD program if the schematic is printed, copied and pasted, though they require some editing in CAD for appearance.

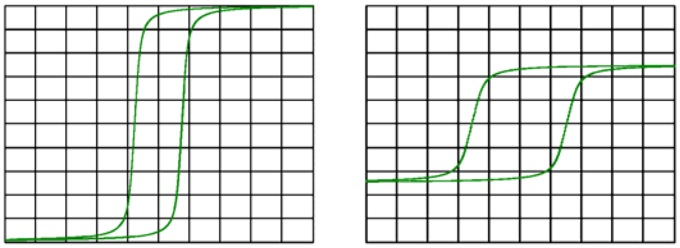


Note that as v(B) approaches Bsat, the coupling factor Kc goes to zero asymptotically. The voltage Vm goes to zero as the coupling factor Kc goes to zero, regardless of the source voltage Vin, so B can never increase beyond Bsat. In a more elaborate SPICE model, Bsat can be a variable, too, the output of a behavioral voltage source, perhaps to include temperature as a parameter.

Although most power converter transformers do not saturate, it is important to include saturation so that flux walking can be detected.

Hysteresis loop:

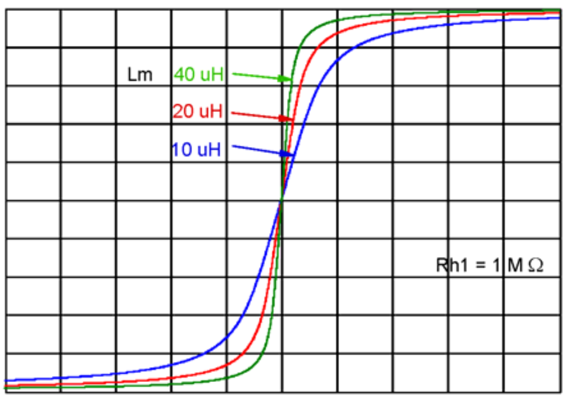
The hysteresis loop is displayed in SPICE using the oscilloscope function. The X axis is the input current, Iin, and the Y axis is the flux or flux density B. The area within the hysteresis loop is the core loss. The hysteresis loop may be copied to a CAD program, where it may be cleaned up and scaled, if necessary, for presentation.



The importance of scaling correctly is illustrated by the above hysteresis loops: They are the SAME hysteresis loop, scaled differently! Successful curve fitting requires comparison of similar curves using the same scale factors.

Curve fitting:

The first parameter to be simulated is the magnetizing inductance. Rh1 is set to a high value, such as 1 MΩ, and Lm is set to the measured or estimated magnetizing inductance for a one turn winding. The figure below shows the magnetizing line as the inductor Lm is varied from 10 uH to 40 uH.

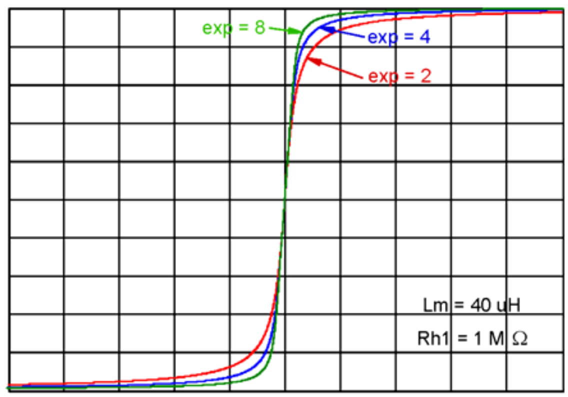


Do not worry about the shape of the corners for now, look only the slope of the line near the X axis. Be sure that the coordinate scale factors are correct or the curves cannot be matched by visual comparison, and slope calculations will be necessary.

The "roundness" of the corners may be adjusted next. This is accomplished by varying the exponent in the expression for *Kc* in the behavioral voltage source B12, with reference to the SPICE model schematic above.

$$Kc = 1 - \left(\frac{B}{Bsat} \right)^{Exponent}$$

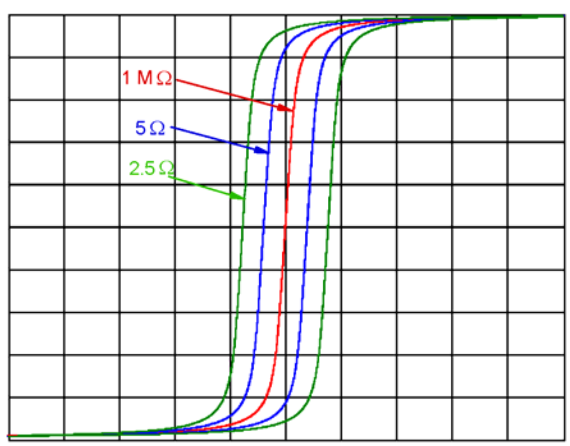
As the exponent is increased, the corners get sharper. The graph below shows exponents of 2, 4 and 8, using the 40 uH curve from the graph above.



Next the core losses are modeled by varying the value of the resistor Rh1. As the resistor R1 is reduced in value, the hysteresis loop opens up.

As the SPICE model is refined, Rh1 will be replaced with a more complex function, but it will be resistive in nature and represents a loss whenever a voltage is applied and the core is not saturated.

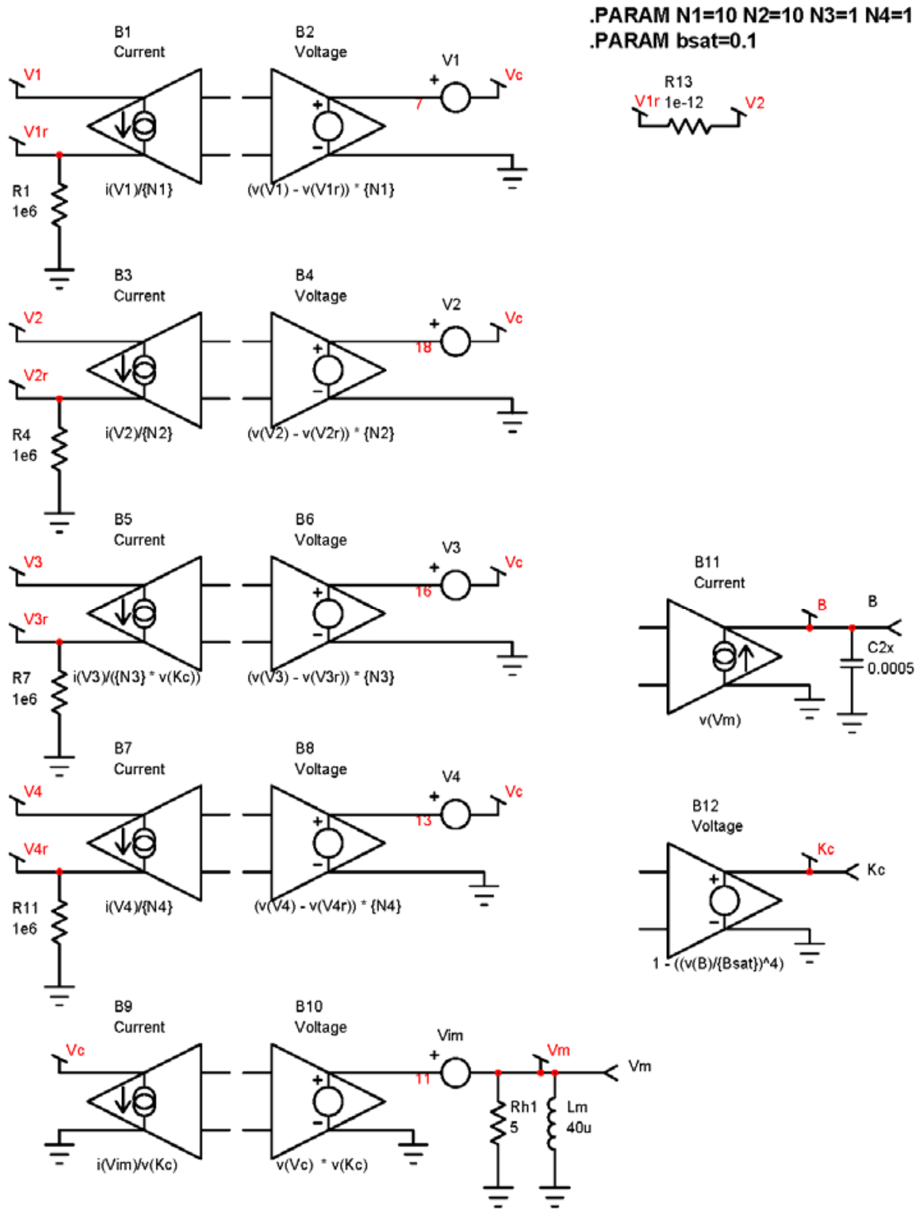
The graph below shows the change in the model hysteresis loop as the value of the resistance Rh1 changes, for Rh1 = 1 MΩ, 5 Ω and 2.5 Ω.



The parameters (L2, the exponent and R1) can then be tweaked to refine the match to be as close a fit as possible. If the shape is not just right, at least ensure that the area enclosed is as close a match as possible.

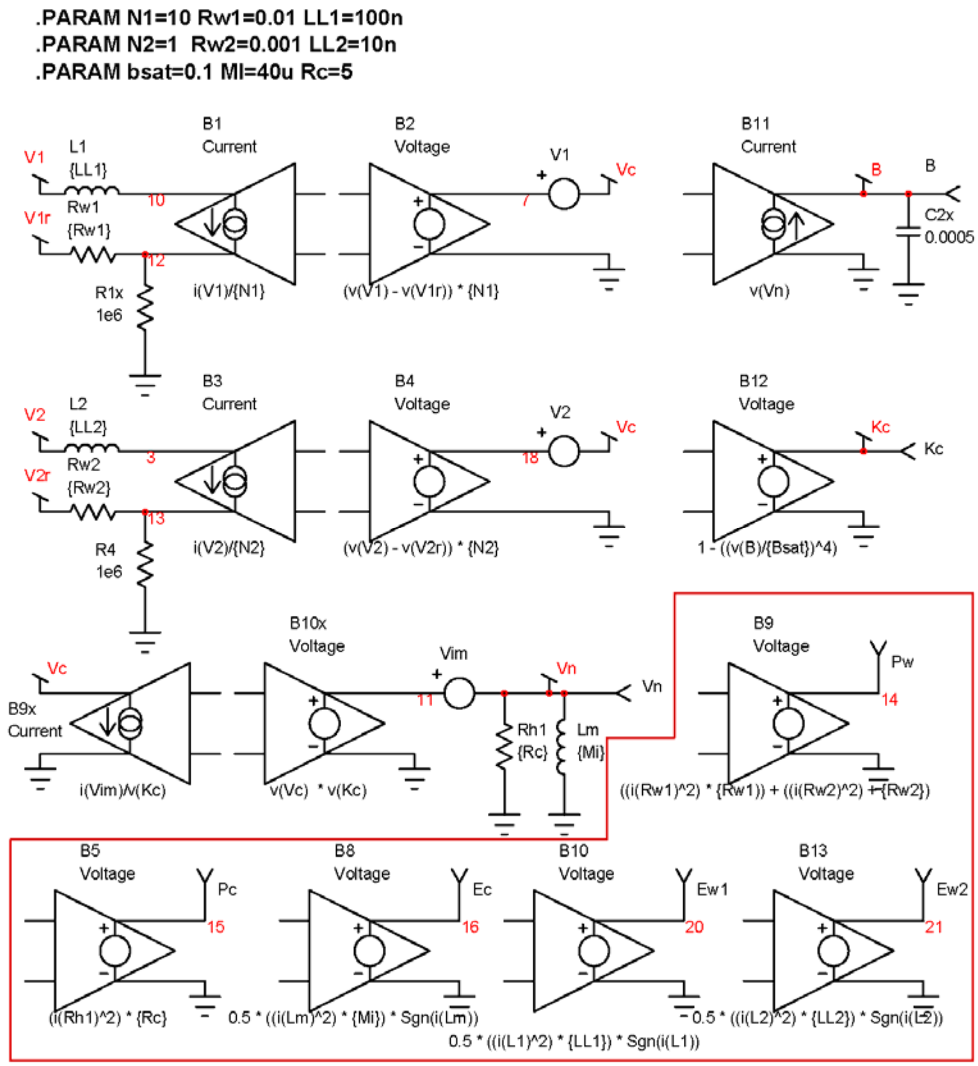
SPICE model, 4 turn transformer with hysteresis and core losses, simple model:

The SPICE model below adds the core loss and hysteresis functions to the ideal four winding transformer presented above. The loss and hysteresis functions join the transformer at the node Vc.



Winding resistance and leakage inductance, simple model:

The SPCE model below is for a two winding transformer with the hysteresis and core losses from the previous example, plus winding resistance and winding leakage inductance. In this example, some of the component values are in SPICE parameter statements, so that they can be changed without editing the schematic. This is discretionary.



Note: This model has not been run and verified, since it does not have a voltage source and loads, so there may be errors in the functions. It is included as an example for discussion and qualitative analysis.

Loss and energy test points:

The test points and the behavioral functions that generate them (red box) are not a functional part of the SPICE model. Their purpose is to provide a display of the transformer's losses and stored energy: Core loss Pc, core energy Ec, winding loss Pw and winding energy Ew1 and Ew2. They are calculated as I^2R and ½ I^2L, and may be summed, if desired, when more than one component has losses or stored energy.

The test points for losses are obvious for a SPICE model that is concerned about efficiency, but the test points for energy require an explanation. In understanding losses in a power converter, it is important to account for stored energy and look for discontinuities and reversals. If the energy changes abruptly, it very likely results in a loss in another component. In the case of transformers, when the transformer switches polarity, the stored energy is very likely to be dissipated in the MOSFETs and attributed to "switching losses". While not entirely incorrect, it is more productive to identify the origin of the losses as it may lead to improvements in efficiency

A transformer may seem to be very efficient, if only the core and winding losses are identified, but a poorly designed transformer that has excessive leakage inductance may cause the overall efficiency of a power converter to be quite poor.

Note that the behavioral equations for the energy test points Ec, Ew1 and Ew2 contain the terms "Sgn(i(Lm))" "Sgn(i(L1))" and "Sgn(i(L2))". The purist may say that this is incorrect, energy is always positive, but I believe that it is important to distinguish the direction of current flow in the energy test points. Thus the magnitude of the signal is the amount of stored energy and the polarity of the signal indicates the direction of current flow. The importance of this is that we are looking for rapid changes in stored energy so that we can account for the energy, and particularly where it goes. If the change is between equal currents of opposite polarity, the transition is much less apparent if the Sgn() term is not used.

More windings can be added to the transformer SPICE model simply by adding additional similar SPICE winding sections. They can be copied and pasted, then edited. I have chosen to give each winding its own SPICE parameter statement, but similar windings, such as the two halves of a push-pull winding, can use the same SPICE parameters.

The SPICE models above are simple models, not including high and low frequency effects except to the extent that they can be lumped into the simple components. However, they will be useful for many applications. Once a model is validated for a particular transformer, it can be converted to a SPICE sub-circuit so that it can be placed as a component into higher order assemblies.

Caution: A SPICE model may have very limited range of usefulness in simulating a real transformer. Be sure that the parameters of the model closely match the simulated conditions.

Core loss, part 2: low and high frequency effects:

Classic theory of magnetic core losses teach that at a given frequency, the maximum flux density $\hat{B}$ determines core losses. If this is true, pulse-width modulation (pwm) for voltage regulation does not increase losses. Twice the voltage for half the time is the same $\hat{B}$, so the losses are the same. Unfortunately, this behavior is a low frequency phenomenon, and most present-day power converters operate at high frequency.

At high frequency, core losses are resistive in nature, that is, proportional to V^2. Twice the voltage has four times the losses. If the duty-cycle is 0.5, then the average losses are one half of four, that is, doubled. The generalized expression for relative average loss at reduced duty-cycle *d* is

$$P_d = \frac{P_1}{d}$$

where P_d is the average power at a reduced duty-cycle *d*, and P_1 is the power with a duty-cycle of 1.0 (or 100 %).

This will be an unpleasant surprise to those who think that pwm is an efficient design.

Hint: To determine which loss model applies, look at the slope of the curve for the loss density at the flux density $\hat{B}$ and frequency of interest in the core material data curves. If the slope on the log-log graph is nearly +2, the losses are resistive. If it is more nearly +3, the more complex low frequency loss model is appropriate. If the slope is an intermediate value, the SPICE model should have mixed characteristics with a frequency-dependant cross-over.

The derivation of the following asymptotic models is in Appendix A. The units are from that derivation, and do not follow MKS standards, so do not apply them generically.

For the low frequency case, the asymptote for the instantaneous power P_L' is

$$P_L' = \frac{E^2}{R} \qquad \text{mW/cm}^3$$

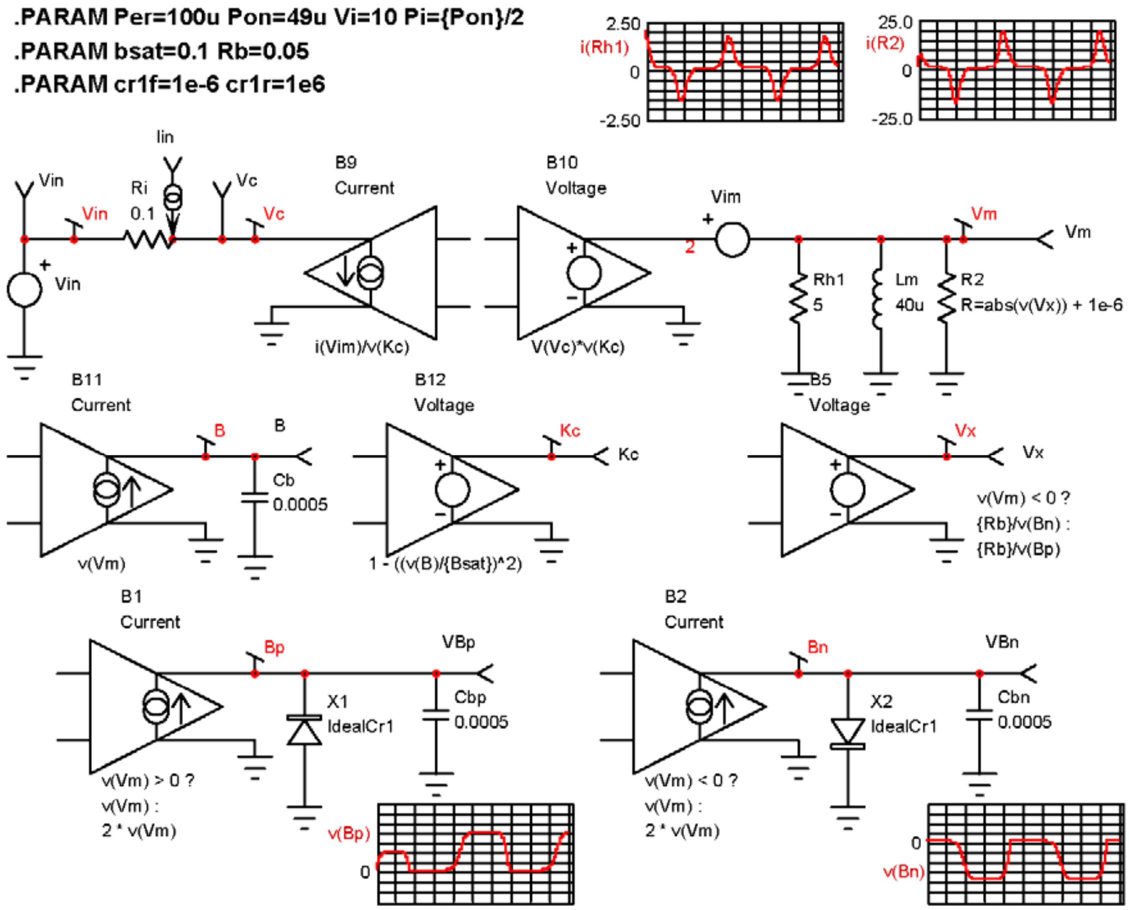
where $R = \dfrac{32}{a * B}$ for $B > 0$.

For the high frequency case, the asymptote for the instantaneous power P_L' is

$$P_L' = \frac{E^2}{R} \qquad \text{mW/cm}^3$$

where $R=256/a$.

The "*a*" is from a Magnetics, Inc. approximation formulae, and it is different for the different frequency ranges. *B* is the instantaneous volt-seconds/turn, flux or flux density depending up the units and scaling for the model. *E* is a voltage per turn density function, with units of volts / turn cm^2, in the example. See Appendix A.

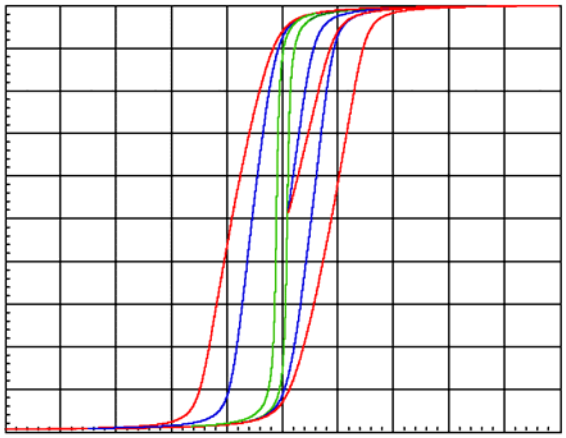


The low frequency function is modeled as a resistor R2 whose value is a function flux, v(Vx), the output of a behavioral voltage source B5. The high frequency model is simply a resistor Rh1. Lm is the magnetizing inductance, as in the simple model.

Generating v(Vx) requires two new flux generators, the behavioral current sources B1 and B2. Both depend upon the square wave being reasonably symmetrical, the positive on time being comparable to the negative on time and the voltage not changing too much, cycle to cycle. About a 2:1 difference in volt-seconds is the limit for the variance, so this condition is not too restrictive.

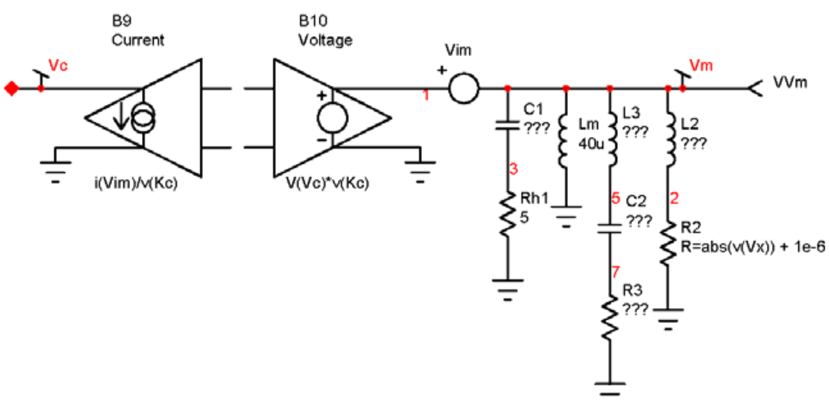
In B1, the current charges the capacitor Cbp to model the volt-seconds since the last negative to positive transition. If the core saturates, the voltage Vm goes to zero, thus the flat top of the flux curve v(Bp) as the conditions modeled go far into saturation. When the voltage reverses, the capacitor is discharged at two times the rate, but it cannot go negative because of the ideal diode subcircuit X1. In the B5 voltage source, the correct voltage VBp or VBn is used depending upon the polarity of Vm. and the function Rb/*B* is generated to control the instantaneous value of the resistor R2.

Curve fitting:



The curve above shows the effect of varying the parameter Rb, which controls the value of R2. The green curve is a baseline, with Rb set to a very high value. The green hysteresis loop is the high frequency model, though it is artificial at the present conditions, which are low frequency conditions. As Rb is decreased to 0.1 Ω, the blue curve results, and the red curve is with Rb decreased further to 0.05Ω. Note that the hysteresis curve fattens as a wedge, getting wider as *B* increases.

The SPICE model above includes the low and high frequency asymptote loss characteristics. It will almost certainly need addition elements, as suggested in the model below, to fill in the loss curve over the frequency range, with low, high and band-pass filters, to tweak the curve. To begin this effort, it will be necessary to have data taken on real components. I suggest a curve of average loss density for log steps of voltage *vs.* log steps of on time for applied ac square waves, and a curve of instantaneous loss density for step functions at log steps of voltage *vs.* log time, taken from negative saturation to positive saturation.



If a SPICE model can reproduce both curves, I believe that it is successful.

High frequency effects in the windings:

Eddy currents:

While all of the "high frequency" effects are due to eddy currents, usual jargon applies the term to losses in the core due to currents induced in the core by the changing flux. These can be modeled as a winding on the core.

My interest is in high frequency transformers for power converters, and they usually use ferrite cores. Everybody "knows" that ferrites do not have eddy currents, as their resistivity is very high. This may be true, but should be revisited. A number of years ago, George Schaller, of Ceramic Magnetics, made two sets of cores for me that were designed to show if there were eddy current losses, and the loss curves fell on top of each other, suggesting that there are no eddy current effects. I cannot remember the details of the design, and perhaps the tests were not run to a high enough frequency.

Ferrites have an extremely high dielectric constant, of the order of 300,000 to 500,000, which may allow reactive currents to flow around the core. The dielectric constant, and thus the capacitance, is a complex number, and the imaginary part is lossy. I remember reading somewhere that there seemed to be eddy currents in ferrites with paths that were larger than a domain but far smaller than the core as a whole. I do not remember the source, but this may explain why the core sets that Ceramic Magnetics made did not show any difference.

Regardless, any eddy current losses in ferrites are lumped into the core losses, and they may explain why the core losses look resistive at very high frequencies. The possibility of eddy currents, which may be geometry dependent, suggest that core loss tests should be performed on the actual core used, not just on a "standard core."

Because eddy current losses can be lumped into the core characteristics, I do not believe that any special provisions are needed in the SPICE model.

Winding losses:

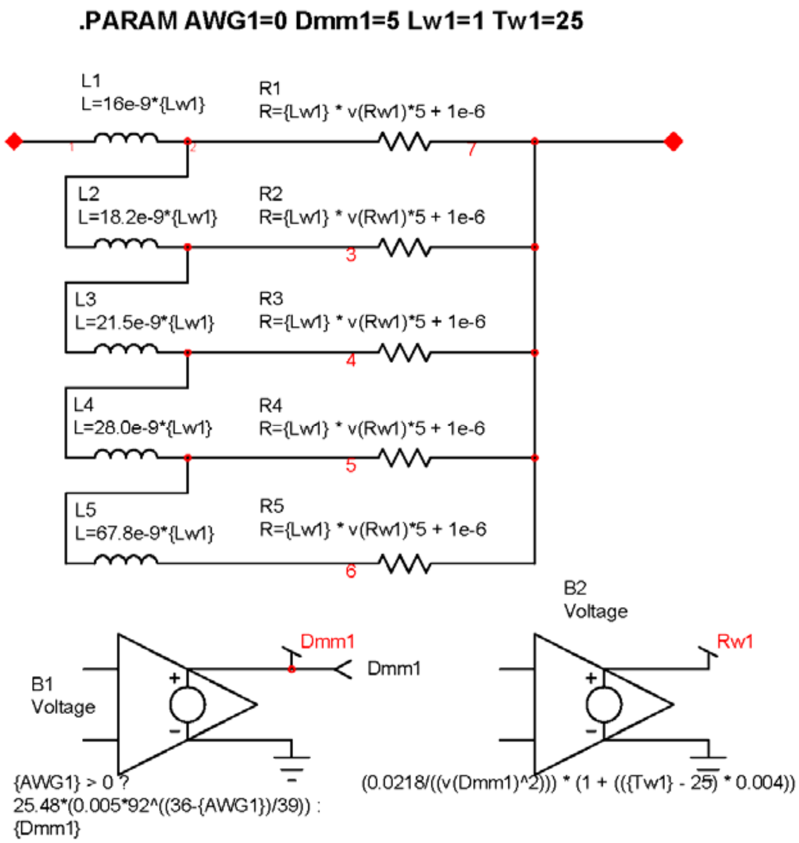
For low frequency transformers, the winding losses can be modeled simply as a resistor with the same value as the resistance of the wire used to make the winding. The model may include temperature as a parameter to correct the resistance for temperature rise.

At high frequency, skin effects and proximity effects are important and may dominate. Both are due to eddy currents, but deserve individual consideration.

Skin effect:

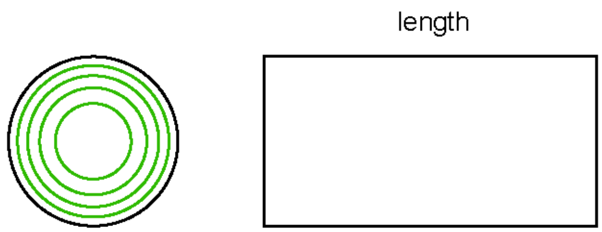
When the current changes in a conductor, eddy currents are induced in the conductor. For a super-conductor, these eddy currents are permanent, and they exclude all magnetic fields. For a conductor having resistance, the eddy currents die out quickly, but while they persist, they result in higher losses. The eddy currents limit the penetration of the increased (or decreased) current into the conductor, forcing the current to flow initially near the surface. This is often described as an increase in the "apparent resistance," sometimes called the "ac resistance".

> High frequency effects should not be modeled as a change in resistance *R*, if the model must be accurate for both voltage drop, $I*R$ and power, $I^{2*}R$. Nor should it be modeled as a change in inductance *L*, or energy $½*I^2*L$ will not be conserved. It must be modeled as a change in current *I* or current distribution.



The SPICE model above simulates a wire, with skin effects. The R-L-R-L (etc.) concept can be found in the literature and on line, but I think that this implementation is easier to use. The parameters are either the AWG gauge {AWG1} or the diameter in mm {Dmm1}, the wire length in meters {Lw1} and the wire temperature in ºC {Tw1}.

In its derivation, a conductor is divided into five concentric segments of equal area, thus equal dc resistance per unit length. For a given round wire size, the dc resistance per unit length of each segment is five times the resistance found in any wire table for the wire size, and that resistance v(Rw1) is calculated in the B2 behavioral voltage source. If the wire size is entered in AWG, the B1 behavioral source converts it to mm, as v(Dmm1). If the temperature is different than 25 ºC, correction for temperature is also calculated.



The inductance is more complicated, but the value of each inductor is the inductance that gives the correct time constant *L/R* for the penetration depth, given the resistance of a layer. It appears to be independent of wire size, at least approximately. For larger wire, the resistance *R* goes down, so the time constant *L/R* goes up. This is an approximation, but it should be sufficiently accurate for most SPICE models. With more analysis and real test data, this model will be refined.

An oscillograph of the current increments in the various layers is shown for a more complex SPICE model later in this presentation. It is interesting that the inner currents may flow in the "wrong" direction after a current transition, and may persist after the source current goes to zero.

Note that the value of the resistances does not change (unless the temperature changes). The current is redistributed by the L-R-L-R (etc.) function. The model will be suitable for both voltage drop, $I*R$ and power I^2*R.

The wire may be converted into a SPICE subcircuit, and placed where the winding resistance is located for each winding in the model for the winding resistance and inductance. The winding leakage inductance can be lumped into the wire model, but it may be preferred to keep them separate.

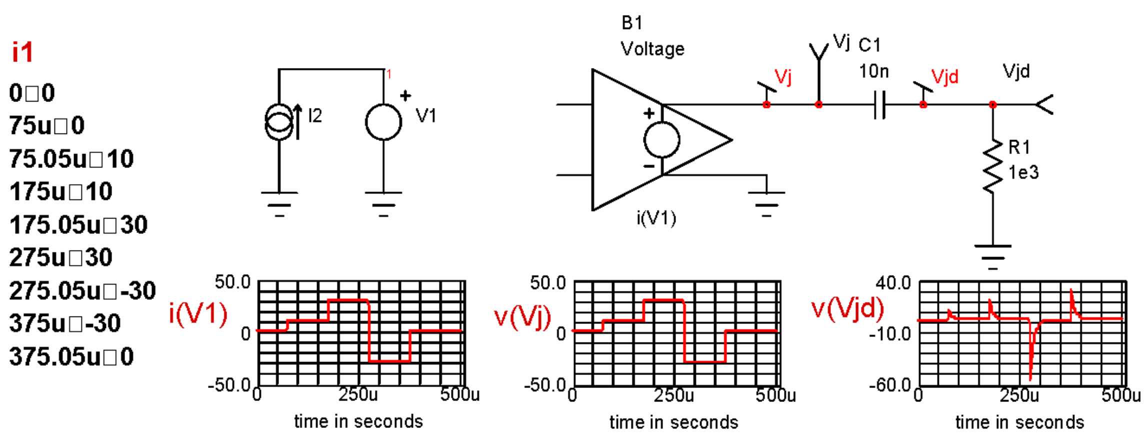
Proximity effects:

The proximity effect is harder to quantify, and there are several loss mechanisms that are lumped under this title:

- The effects of current crowding due to the current in adjacent conductors.
- Apparent current multiplication in successive layers of a multi-layer winding.
- Eddy current losses in adjacent conductors (windings, shields).

All are dependent upon di/dt in the winding, but the effect persists long beyond a step change in current, so a function of di/dt alone will not suffice in the time domain. For sine excitation, di/dt is a well behaved cosine term that is useful for ac analysis. A SPICE model requires that the losses be modeled in the time domain.

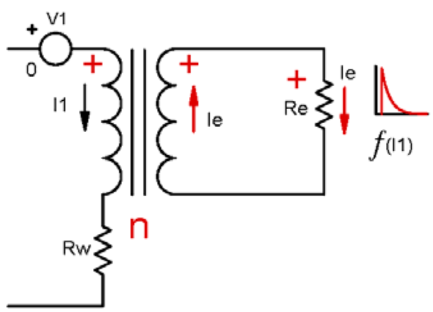
An understanding of the loss mechanism points the way. A step change of current induces an equal and opposite current in nearby conductors if the coupling is perfect. So, the first step is to model a current equal to the difference current of the step change. This current flows through a resistance, causing losses, and this current and the associated losses decay exponentially with time.



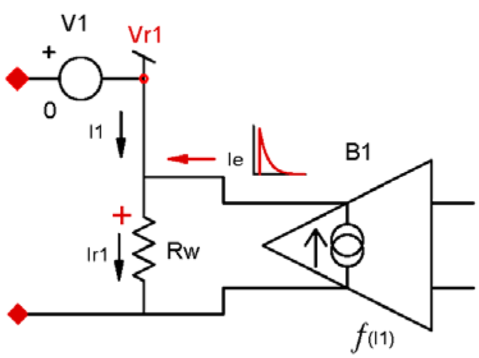
The SPICE components above generate a waveform of the required shape. A current generator I2 is programmed using the PWL function as in the chart. Note the 50 ns rise times. The zero volt voltage source V1 is used to measure the current. The behavioral voltage source B1 produces a voltage Vj that is the analog of the current. The high pass filter C1, R1 produce the voltage Vjd that has the desired shape. If the time profile isn't quite right, a different filter circuit can be used, and the signal v(Vjd) can be scaled to any magnitude and combined with other functions in a behavioral source.

Traditionally, high frequency effects are described in terms of an increase in the apparent resistance. That would be easy to implement, simply increase the resistance as a function of Vjd. The problem with varying the resistance is that the SPICE model must use the function to model both the correct voltage drop and the correct losses due to the eddy currents. The voltage drop is IR, and the power is I^2R. For both of the voltage drop and power equations to work, it is the current I that must change, not the resistance R.

The eddy currents are generated and coupled back to the wire through an equivalent coupled inductor, as shown below. However, I do not know what is in the SPICE coupled inductor model, and it seemed to give anomalous and unpredictable results. A transformer model could be used, incorporating inductors, but this approach rapidly became quite complex.



The schematic at the left shows an eddy current Ie, generated by the coupled inductors as a function f(I1) of the step current in the wire. The eddy current Ie causes a voltage drop when it passes through the resistor Re. The model for the resistor Re should also reflect the skin effects. Likely, it is the same or similar to Rw, the resistance of the wire carrying the current I1 that induced the current Ie, or some portion of it. Repeating that model as the load of a transformer circuit, while more rigorous, is awfully complex.

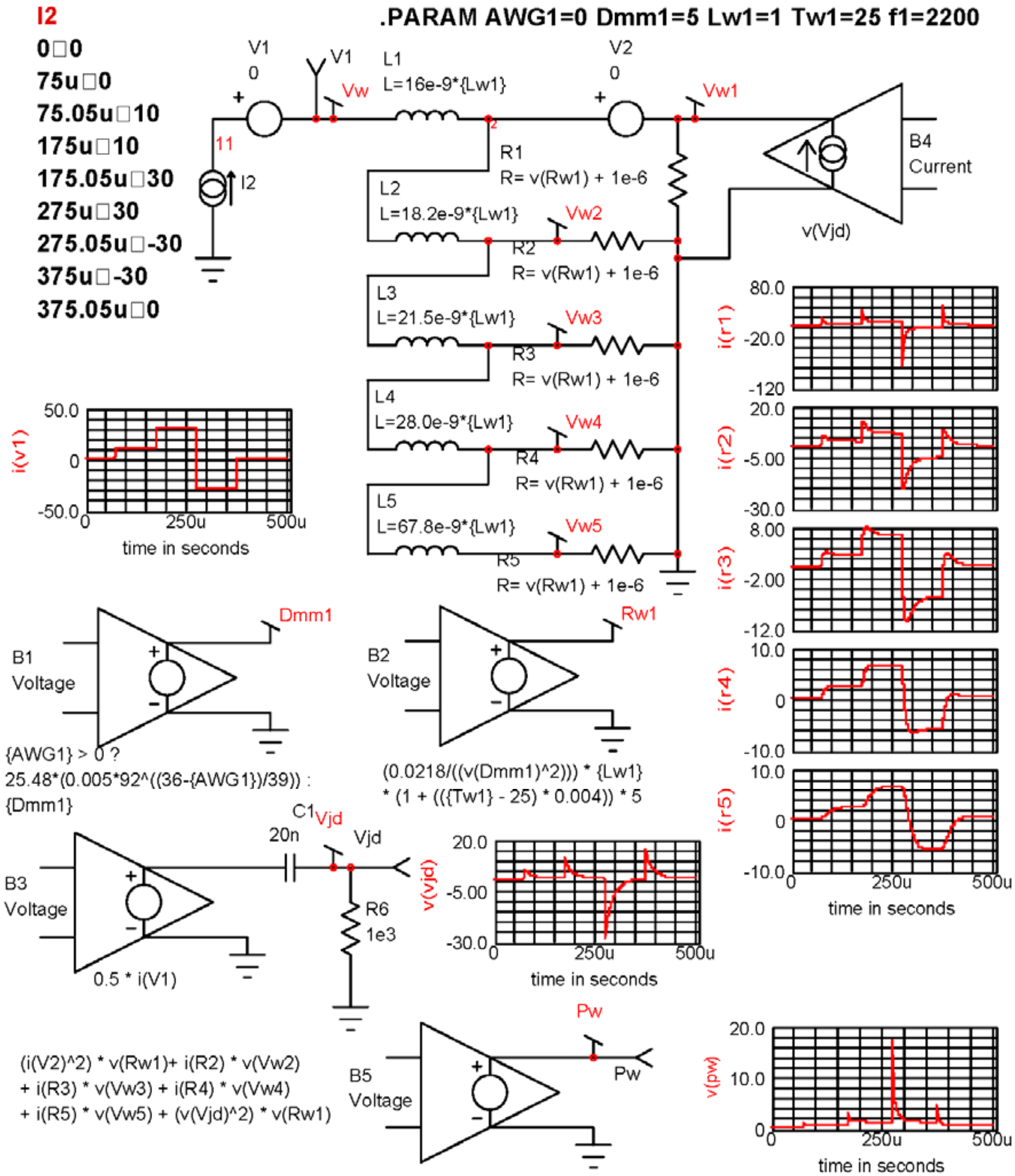


The SPICE model at the left can approximate this simulation, and it is very simple. If n is less than 1, that can be simulated as a linear scale factor in f(I1) in the behavioral current source equation.

This model provides the correct voltage drop at Vr1, $Rw * (Ir1 + Ie)$. However, since this circuit is a surrogate for the circuit above, the power in Rw requires careful interpretation. It is NOT $(I1 + Ire)^2 * Rw$ or $Ir1^2 * Rw$. The circuit is designed to reflect the correct voltage drop to the rest of the circuit, and it depends upon the assumption that the wire resistance, even with ac effects, is small compared to the load and other resistances in the circuit. To calculate the power for the winding loss test point, the correct formula is given by the equation $Pw = (I1^2 + Ie^2) * Rw$. This is because in the more rigorous model, there are two resistors, and the losses (power) must be calculated in the simple model as if it were two separate resistors equal to Rw each with its own current.

The SPICE model for a wire, described above, with modification, is suitable for Rw, as it uses unvarying resistors, with L-R filters to redistribute the currents for di/dt. To add proximity effects, a behavioral current source is applied to the resistor that represents the outer layer of the conductor, on the premise that eddy current losses occur near the surface.

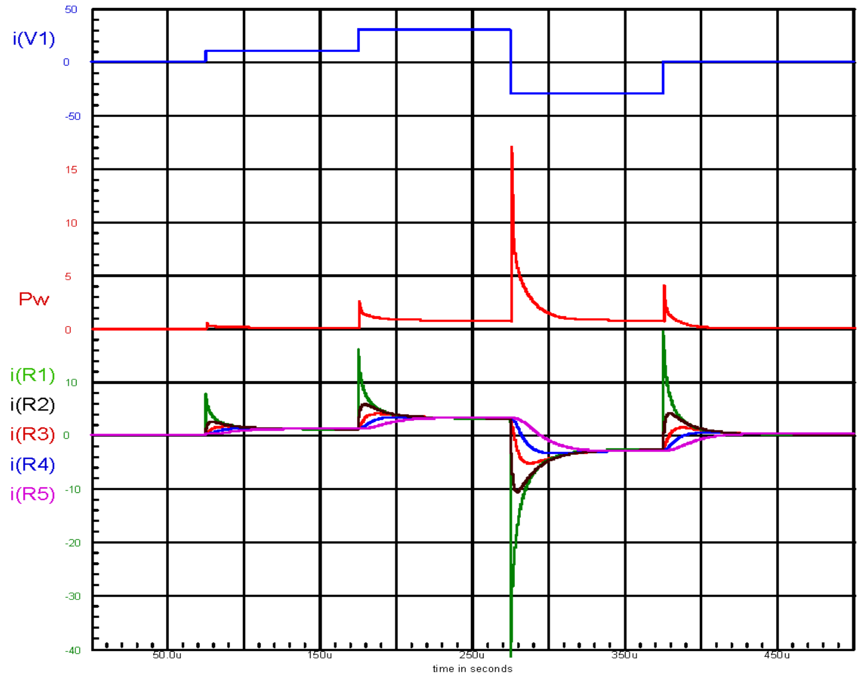
The SPICE model below is for a wire with proximity effects added. A behavioral voltage source has been added to calculate the losses in the wire as sum of the products of the voltage across each resistor and the current through it. Note the spikes in power following a step in current, either direction. No attempt was made to scale the parameters correctly at this time, it is an example showing the SPICE building blocks for qualitative discussion.



Spice model of wire with proximity effects

A simple model is preferred to one that is rigorous, as long as it can model the functions and parameters seen and measured in a real transformer. It is acceptable and necessary to tweak the parameters of the models as "fudge factors" to achieve this result.

The graph following shows the currents (input current i(V1), the currents in the shells of the wire, i(R1), i(R2), i(R3), i(R4) and i(R5), and winding loss Pw using the SPICE oscilloscope function. It was copied to a CAD program and edited for appearance.



Note that following a reversal of current in the wire, the currents in the core of the wire may continue to flow in the wrong direction for a significant time. Note also that eddy currents, and their losses, persist beyond the time that the input current goes to zero. With zero current flow, a model that changed the "apparent resistance" by varying the value of a resistor could not show this effect—with 0 current, the model would show 0 power regardless of the value of the resistor.

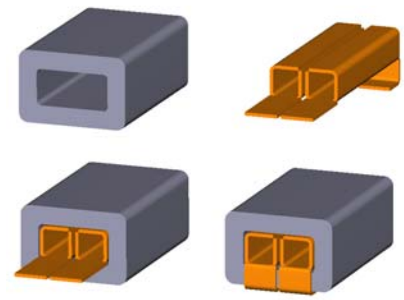
No representation is made that these values are accurate; they are for qualitative analysis only. While some liberties were taken to make a simpler SPICE model of losses due to the skin effect and proximity effects, I believe that it is adequate for most transformers.

Multi-layer windings:

This model should also be suitable for the layers of a multiple layer coil. Each layer should be modeled as a separate wire, with all layers in series. The injected current for each layer should have the same timing, but the magnitude of the injected current will be much larger for each successive layer.

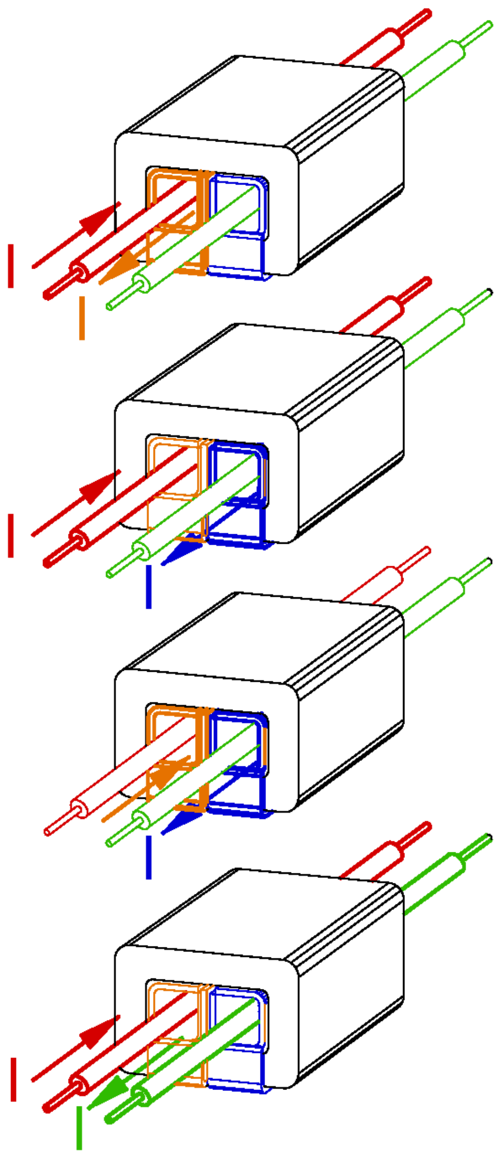
A model for eddy currents induced in a shield or another winding will likely affect only the layer adjacent to the shield or the other winding. The loss should be reflected to the winding causing the eddy-currents, not the shield or other winding in which the eddy currents are induced, to be consistent with the model above. I am not trying to cover all possible combinations of eddy currents at this time.

Transformers with coaxial or interleaved windings:



The secondary of a coaxial transformer is made of formed copper, inserted into a magnetic core. A number of similar assemblies are placed with their through holes aligned and the primary winding is then threaded through. A transformer with an n : 1 turns ratio has n assemblies, or "elements" with a single turn primary winding. The secondaries are wired in parallel, essentially constituting a fractional turn of winding 1/n turns.

The leakage inductance of the secondary windings is very low, and when they are used in parallel, the inductance is effectively reduced further. Theoretically, it is reduced by 1/n, being paralleled inductances, but there will be some leakage inductance in the interconnection even if it is planes in a printed wiring board.



For more information on coaxial transformers, see "Coaxial Push-Pull Transformers", a pdf file at http://fmtt.com/.

Because the primary winding and the secondary winding on the same side of the transformer are coaxial, as shown by the current flow in the top figure on the left, the leakage inductance from a primary winding to its coaxial secondary winding is extremely low.

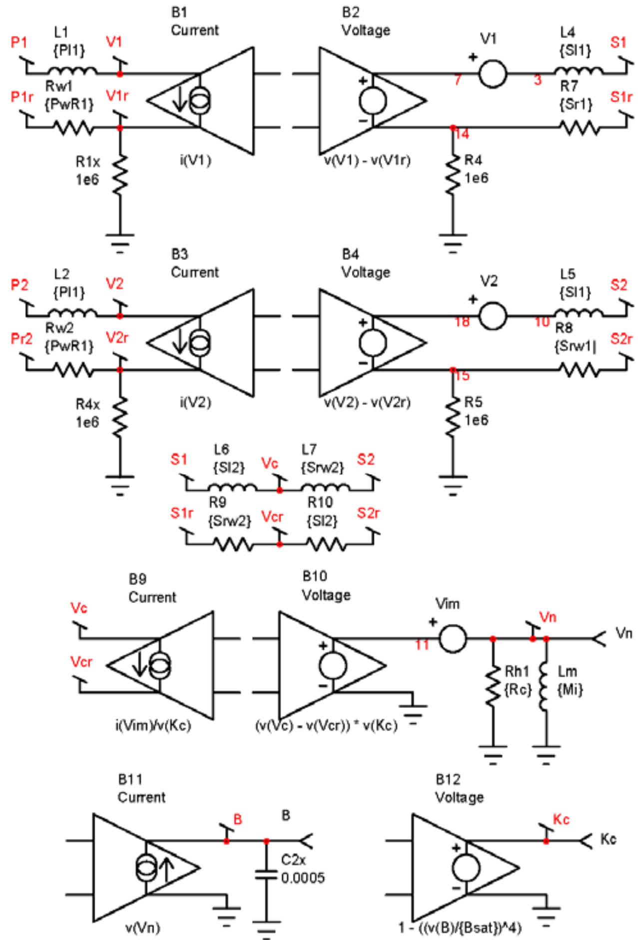
However, the leakage inductance to the other secondary winding, as shown in the second figure on the left, is much greater.

The leakage inductance from secondary to secondary is low, as the windings have flat parallel conductors close together in the center, but it is not nearly as low as a primary to its coaxial secondary.

Finally, the leakage inductance from primary to primary is relatively quite high.

A similar disparity in leakage inductances is true for an interleaved transformer.

While the SPICE transformer model shown at the beginning of this paper is straightforward and easy to understand, it is not adequate to model the special properties of coaxial and interleaved transformers in critical applications. For this, the following SPICE model is suggested:



The behavioral current and voltage sources B1 and B2 are an ideal 1:1 transformer, and model one primary winding to its coaxial secondary winding. Similarly, the behavioral current and voltage sources B3 and B4 model the other primary winding to its coaxial secondary winding. Each has winding resistance and winding leakage inductance defined, and they are very low for coaxial (or interleaved) windings.

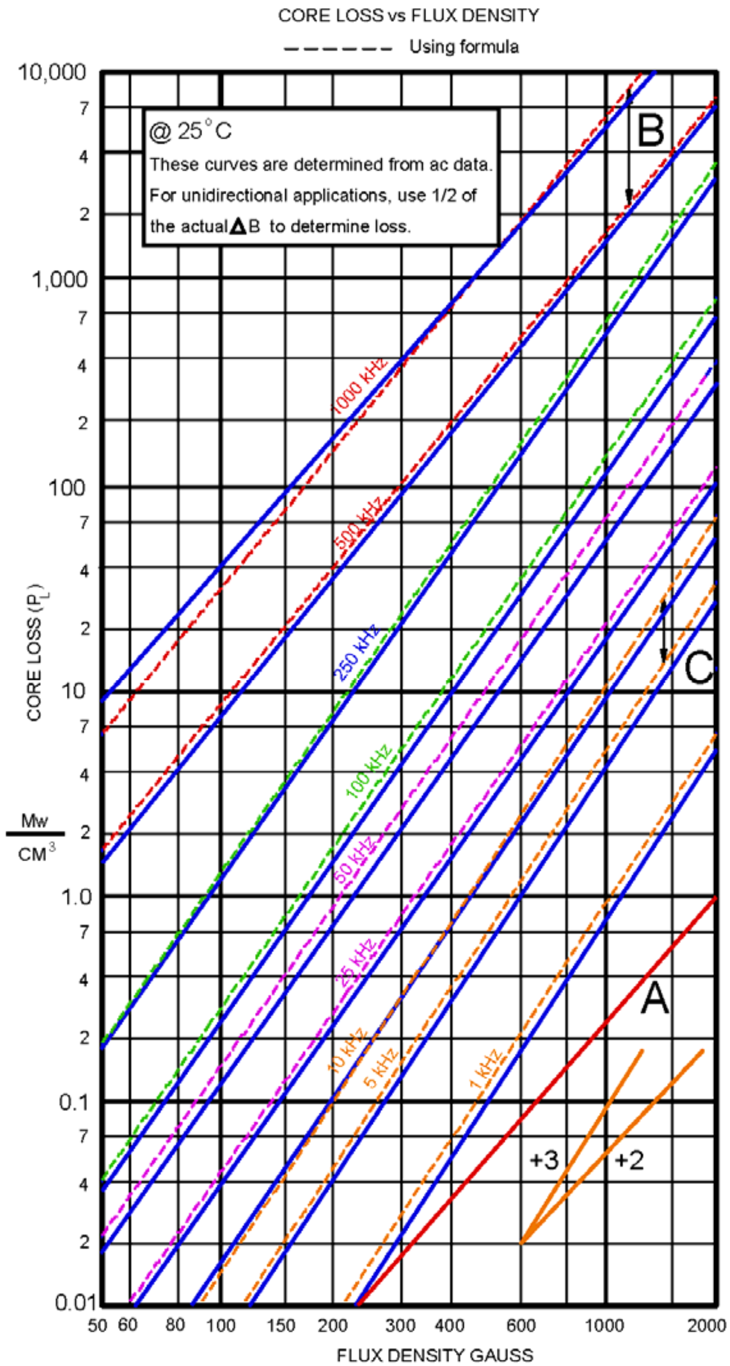
Next, the secondary windings are coupled with a leakage inductance and a winding resistance, to represent the respective leakage inductance and winding resistance from one secondary winding to the other. The resistance will be very low, but there will be significant leakage inductance. For balance, each is center-tapped at nodes Vc and Vcr. The "core winding", representing the core losses, attaches to these nodes.

In any transformer, the relationships between the windings should be reviewed, to see if the simple model can be improved similarly if high frequency effects are important. Interwinding capacitance may be important, and capacitors can be added between the appropriate nodes. This is an untested model, as an example for discussion only.

Appendix A

Core losses in SPICE models from core manufacturers' data:

Loss curves and expressions are usually defined in terms of the maximum flux density $\hat{B}$ and frequency. $\hat{B}$ cannot be determined until the end of the switching cycle, thus it is not an instantaneous term and is not useful for the SPICE model.



Magnetics, Inc. material F

Most manufacturers' core loss data are presented graphically. Occasionally, loss expression approximation equations are given. See Magnetics Inc., Technical Bulletin FC-S7, "Curve Fit Equations for Ferrite Materials." 1999, as an example. The approximations appear to be quite accurate close to the points that were used to derive the approximation parameters, but they appear to be quite bad at other operating conditions.

Magnetic, Inc.'s loss expression approximation is:

$$P_L = a * f^c * \hat{B}^d \qquad \text{mW/cm}^3$$

Where a, c and d are constants, f is in kHz and $\hat{B}$ is in kG.

For Magnetics Inc.'s F material, the constants are given as follows.

Range	*a*	*c*	*d*
f ≤ 10 kHz	0.790	1.06	2.85
10 kHz < f < 100 kHz	0.0717	1.72	2.66
100 kHz ≤ f < 500 kHz	0.0573	1.66	2.68
f ≥ 500 kHz	0.0126	1.88	2.29

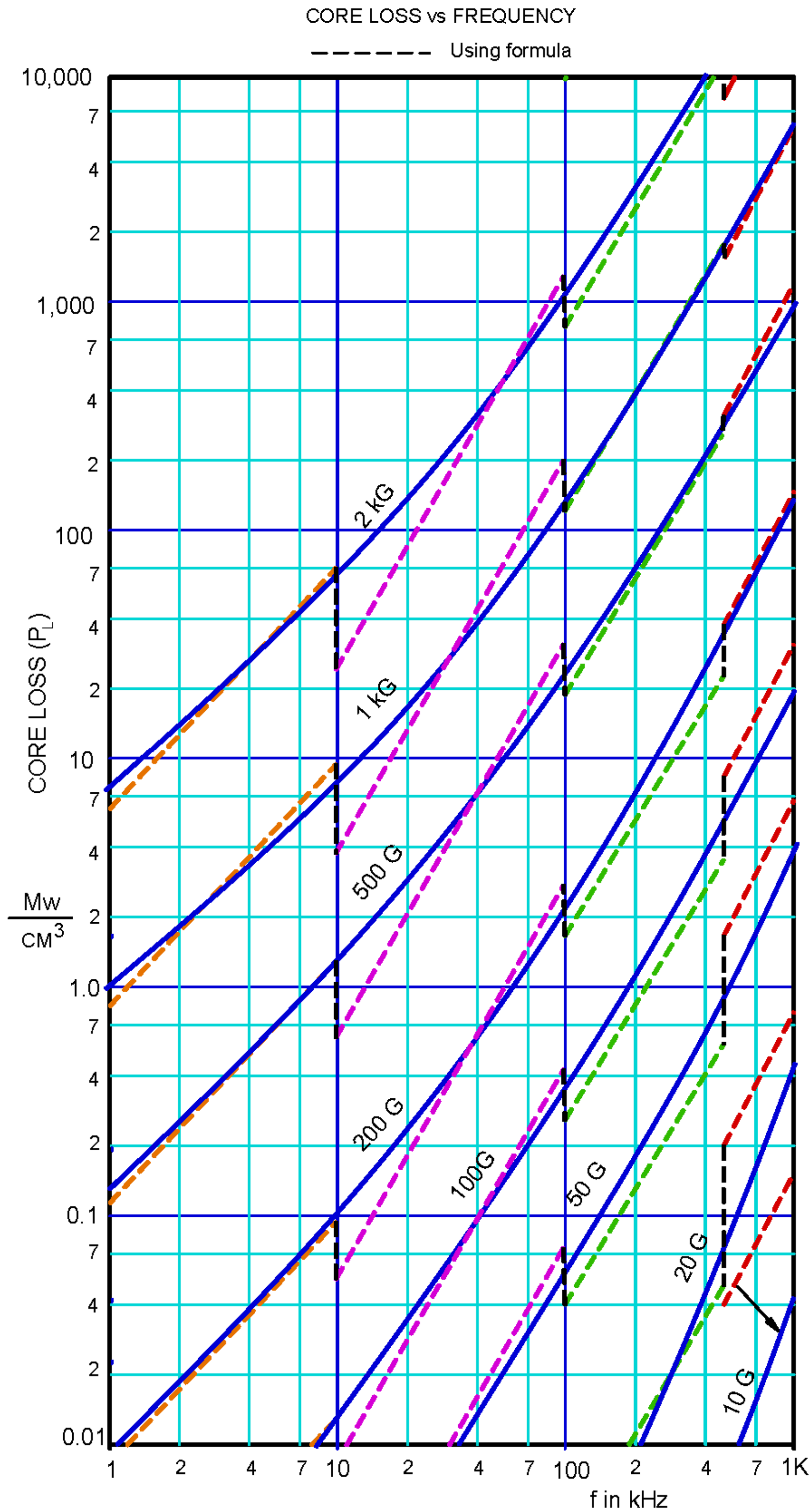
(The colors key to the calculated loss curves in the graphs above and following).

A representative core loss vs. flux density curve is shown above. The basic curves, shown in blue, are for Magnetics Inc.'s F material, transcribed from a Magnetics Inc. data sheet that was found "on line."

Close to each of the lines representing Magnetics Inc.'s F material there is a dashed line in color, showing the results of the core loss approximations using the formula above and the parameters from the table above. Considering the log-log scales, some of the errors are substantial, and I suspect that the base-line data itself is not very good at the extremes. Real data tends not to be such perfect straight lines.

Occasionally, core loss data are presented as curves of peak flux $\hat{B}$ versus frequency. I could not find this data for Magnetic, Inc.'s F material, but the core loss equations can be solved presented graphically that way by sweeping the frequency F for step increments of $\hat{B}$. The result is the graph below, in which the solutions of the equations are the dashed lines, colored to correspond with the frequency range in the parameter table. The solid blue lines are arbitrary curves connecting the average values as best that I could.

The discontinuities in the solutions suggest how bad the approximations are at the limits of each range. In most cases, the continuation is evident, but in the lower right corner, the curve for 10 G looks as if it connects with the curve for 20 G, so I added dashed black lines to show the connections across the discontinuities. The solid blue 10 G line could not rationally relate to any of the solutions, so I placed it arbitrarily, sort of parallel to the 20 G line.



Magnetic, Inc. material F

Parsing the manufacturers' data for SPICE-friendly parameters:

To be useful in SPICE, a model for a transformer must use the parameters which are available in SPICE, principally the voltages and currents of the transformer circuit. A more complex model may have other parametric inputs that affect the analysis, such as temperature. Parameters that cannot be know until after a cycle is finished are less useful and should be avoided, such as frequency and the maximum flux density $\hat{B}$.

Parsing out an algorithm that is restricted to voltages and currents over time from the manufacturers' core loss data is challenging and certainly will not produce accurate results. However, a qualitative understanding of the losses may lead to new SPICE models that embody useful relationships in the time domain. Then, given test specimens and appropriate data for the new models, quantitative analysis will be possible.

Core losses are a function of the voltages in the circuit (including the instantaneous flux density *B*, the integral of voltage with respect to time, an easily derived function in SPICE). Winding current does not enter into the core loss equation. Provided that there is sufficient current to excite the core, currents in the windings can be ignored for calculating and modeling core losses.

Similarly, the winding losses are a function of the currents in the circuit. Voltage does not enter into the winding loss equations. Provided that there is sufficient voltage to overcome the circuit impedances and cause the magnetizing current to flow, voltages can be ignored for calculating and modeling winding losses.

The loss mechanisms at low frequency are different than those at high frequency, and many assumptions about transformer design are based upon low frequency transformer design. Many of the problems of high frequency transformer design may have their root in incorrectly applying low frequency loss mechanisms to high frequency designs, particularly when pulse width modulation is used.

In the graph of loss *vs.* $\hat{B}$ for differing frequencies above (the first graph in this appendix), notice the slopes of the lines. Because it is a log-log graph, the slopes of the lines are the exponent of the function, in this case, the exponent of the $\hat{B}$ function, the parameter *d* in the core loss approximation equations. The highest frequency line, 1000 kHz, has a much lower slope than the lowest frequency line, 1 kHz. This is reflected in the value of *d* in the chart above, 2.85 for the lowest frequency range and 2.29 for the highest. To make this more apparent, see the red line "A" on the graph near the 1 kHz line, which has the same slope as the 1000 kHz data. Also see the two amber lines with slopes of +3 and +2, for comparison.

Notice also the spacing of the lines. Being a log-log graph, the spacing is a function of the exponent *c* of the frequency parameter *f* in the loss approximation, 1.06 for the lowest frequency range and 1.88 for the highest. This is shown by the arrows "B" and "C". "B" is the vertical space between 500 kHz and 1000 kHz, and "C" is the vertical space between 5 kHz and 10 kHz.

For low frequencies, the approximation curve is nearly

$$P_L = a * f^1 * \hat{B}^3 \qquad \text{mW/cm}^3$$

For high frequencies, it is more nearly

$$P_L = a * f^2 * \hat{B}^2 \qquad \text{mW/cm}^3$$

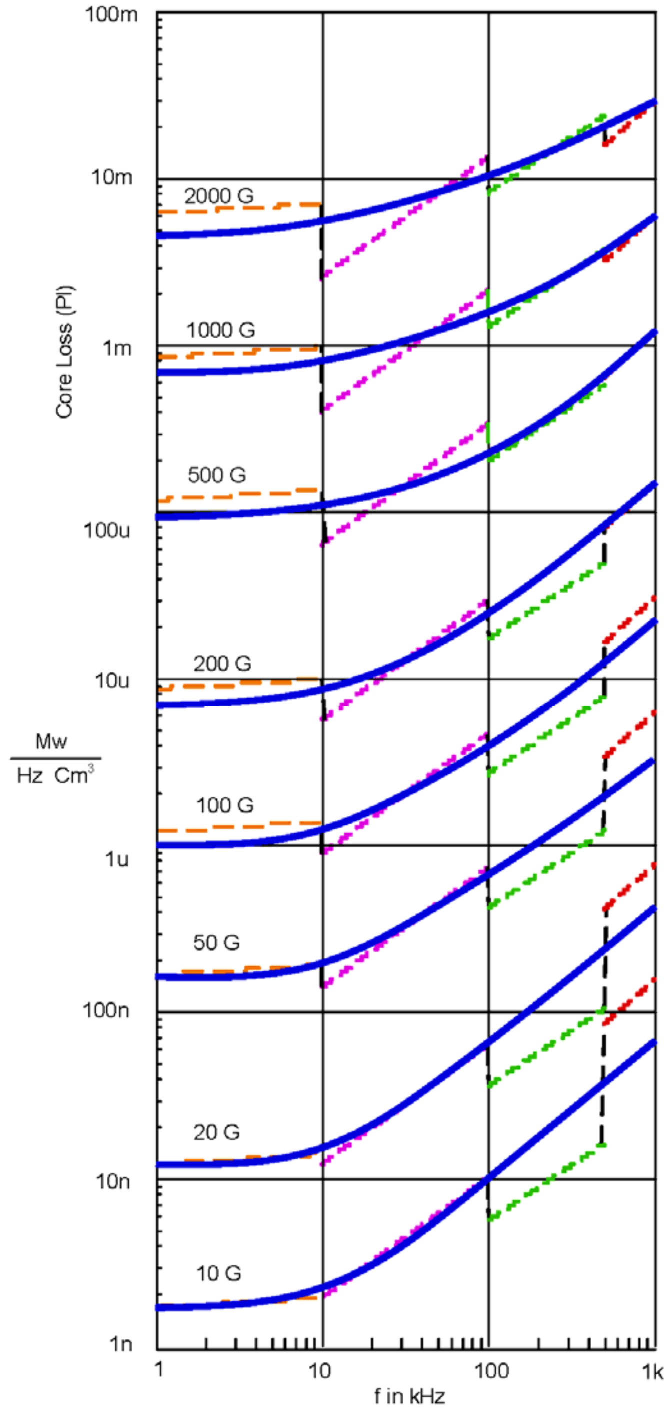
These asymptotes confirm that there is a different loss mechanism at high frequencies.

It is taught and generally accepted that the energy expended to go around a hysteresis loop is constant as long as $\hat{B}$ is constant. In more advanced treatments, we learn that the hysteresis loop may be fatter at high frequencies, an effect that is sometimes attributed to "eddy currents." It is also taught and generally accepted that ferrites do not have eddy currents. Taken to its logical conclusion, this "proves" that if pulse width modulation is used to adjust for input voltage changes, the $\hat{B}$ remains constant and so do the core losses.

Unfortunately, this is not so. However, quantifying the actual losses and predicting them has been daunting.

The graph following uses the same equations as the graph above showing core loss *vs.* frequency, but the result is divided through by the frequency in Hz. The loss units are shown as mW/Hz cm^3, for consistency with the previous graph, but mW/Hz will be recognized as the product of power and time (t = 1/f), milliwatt-seconds, or millijoules. The graph thus shows the energy to go around the hysteresis loop with a constant $\hat{B}$ at different frequencies. As before, the dashed colored lines are the results of the calculations using the Magnetics, Inc. approximation formula and parameters. The solid blue lines are my attempt to smooth the curves. There is no representation of any accuracy at all, but it may be useful as a qualitative analysis.

At lower frequencies, it can be seen that the energy remains fairly constant if $\hat{B}$ is constant, but above 10 kHz, the energy per cycle rises a lot. *<u>This confirms that the core loss mechanisms are different at higher frequencies, and the classic relationship is not useful in the higher frequency range used by many modern power converters.</u>*



Part of the problem of applying manufacturers' core loss data to power converter design is that it is taken using sine wave excitation. Power converters use square wave ac or pulse width modulated square wave ac pulses. Anecdotal evidence suggests that there is at least a qualitative relationship between sine wave excitation and square wave excitation and that the losses are lower using square waves.

Converting $\hat{B}$ to applied voltage is possible with square wave excitation, given the frequency, as the units of $\hat{B}$ are a constant times volt-seconds, where the time is half the period, *1/2F*. Dividing by seconds leaves the voltage function. In other words, knowing the maximum flux density $\hat{B}$ and the time period over which the flux built up (one half cycle), the applied voltage necessary to reach that $\hat{B}$ can be determined.

In the Magnetics Inc. approximations, $\hat{B}$ is given in kilogauss, and *F* is in kilohertz,. Therefore, the staring point is the magnetic equation for voltage in CGS units.

$$E = 4 * \hat{B} * Ac * F * N * 10^{-8}$$

Solving for $\hat{B}$:

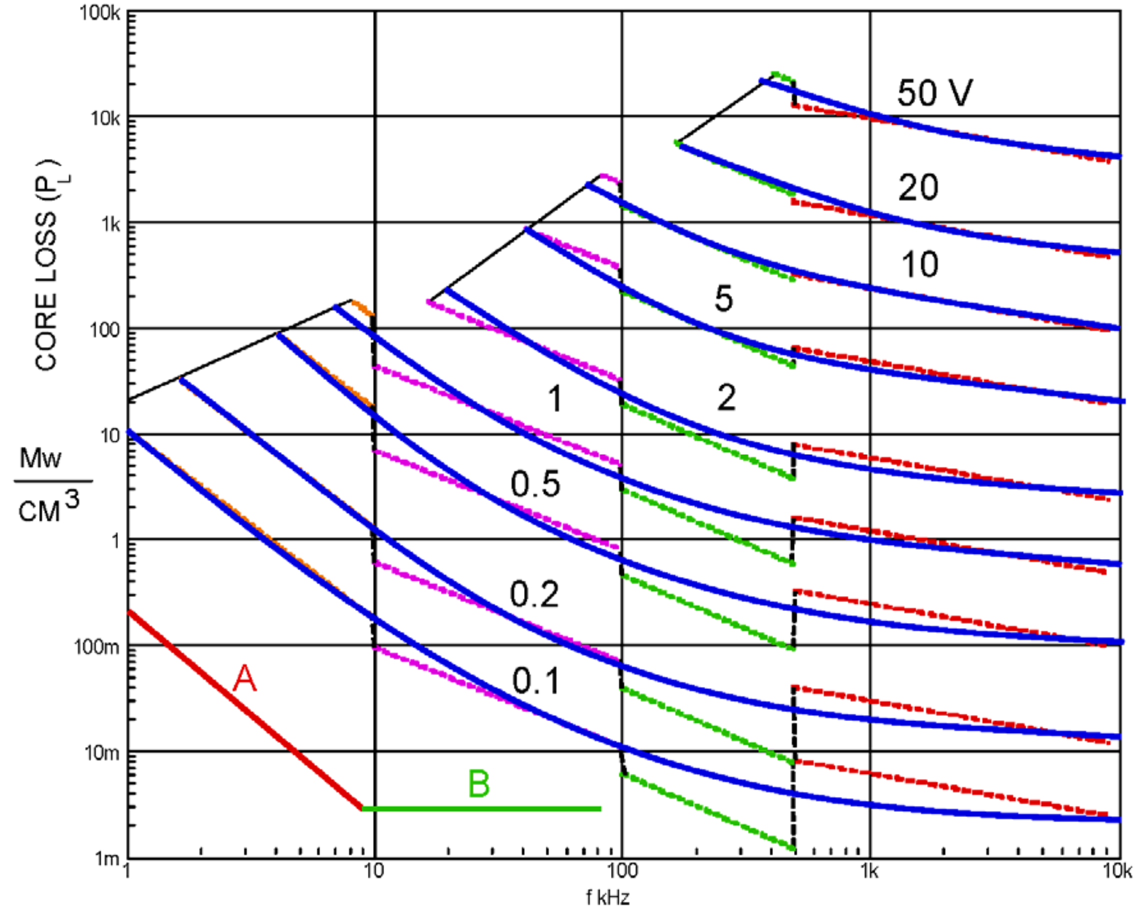
$$\hat{B} = \frac{E}{4 * Ac * F * N} * 10^{8} \text{ gauss}$$

Let N = 1, so the voltage is in Volts/turn, and let Ac = 1 cm^2. $\hat{B}$, in kilogauss with *F* in kilohertz, becomes:

$$\hat{B} = \frac{E}{4 * F_{kHz}} * 10^{2} \text{ kilogauss}$$

Plugging that into the Magnetics, Inc. approximation should yield curves that qualitatively approximate the loss *vs.* frequency for various applied square wave voltages. Because the equations don't know about saturation, the results for low frequency are unbounded, so I added a function to blank the result if $\hat{B}$ is greater than three kilogauss. The lines in the upper left corner show the three kilogauss limit, and they would be one continuous line if the approximations were precise. The X axis is taken to 10 MHz, but there is no illusion of any validity at that frequency. The graphs were extended to better show the changing slope with frequency, and in particular, that it is tending to go flat.

This graph is interpreted as the loss per cm^3 versus frequency with a constant voltage square wave excitation, with a 1 turn winding and a 1 cm^2 core area. Thus the curves are for constant volts per turn density, V/N cm^2. With 100 volts applied to a 10 turn winding on a core having an area of 2 cm^2, use the 5 V line (100 V / 10 * 2). The core loss for the frequency on the X axis is the result in mW/cm^3 read from the graph, multiplied by the core volume in cm^3.



Volts per turn cm^2.

Notice the short lines A and B. The positions and coordinates are arbitrary, but their slopes have significance. They are segments of graphs plotted for the two equations from above, which noted the trends of the exponents of the Magnetics, Inc. approximation equations at low and high frequency.

For low frequencies, the approximation curve is nearly

A $P_L = a * f^1 * \hat{B}^3$ mW/cm^3

For high frequencies, it is more nearly

B $P_L = a * f^2 * \hat{B}^2$ mW/cm^3

The latter is a straight horizontal line, which is the same as for a fixed value resistor. A look at the equation for $\hat{B}$ and plugging $\hat{B}^2$ into the second loss approximation equation B shows why.

$$\hat{B} = \frac{E}{4 * F_{kHz}} * 10^2$$

$$\hat{B}^2 = \frac{E^2}{16 * F_{kHz}^2} * 10^4$$

When this expression for $\hat{B}$ is plugged into the power loss equation B, F_{kHz}^2 is the same as f^2, so they cancel, leaving

B $$P_L = \frac{a}{16} * E^2 * 10^4$$

Thus the losses are independent of frequency, and a resistor may therefore be an appropriate model for core losses at very high frequencies.

Applying similar substitutions for the low frequency model:

A $$P_L = a * f^1 * \hat{B}^3 \qquad \text{mW/cm}^3$$

Solve for $\hat{B}^3$:

$$\hat{B}^3 = \frac{E^3}{64 * F_{kHz}^3} * 10^6$$

F_{kHz} is the same as f , so the f^1 is cancelled, leaving

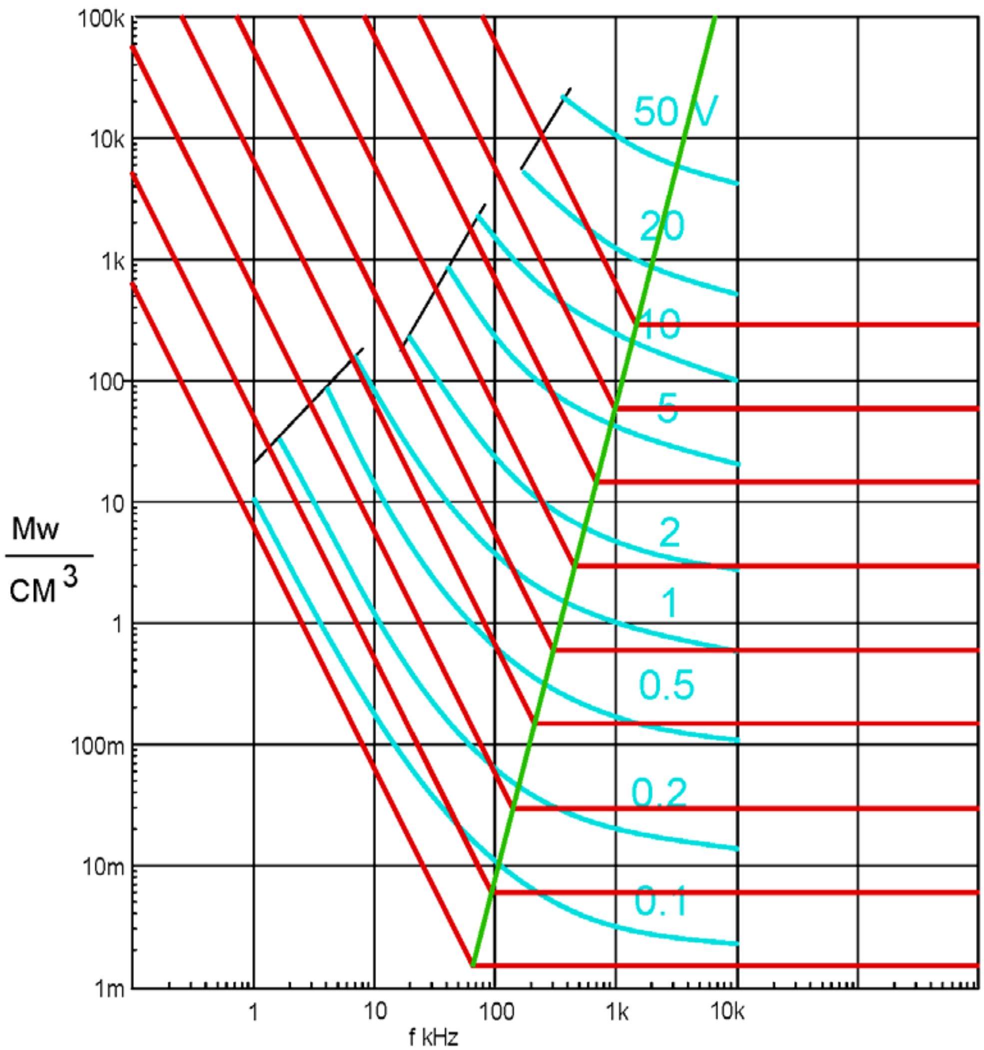
A $$P_L = \frac{a}{64} * \frac{E^3}{F_{kHz}^2} * 10^4$$

Returning to the graph above, in view of these equations, we can begin to parse the asymptotes. For the high frequency end, the spacing for the various voltages vertically should be as E^2, so each increment of 10 x in voltage should be two decades apart. This is not really what is shown, but it is very roughly the pattern, if one visualizes the curves extending further.

For the low frequency end, the slope is determined by the F_{kHz}^2 in the denominator, so the slope -2 on the log-log graph.

The line spacing is determined by the E^3 in the numerator, Pl increases 3 decades for each voltage increment of 10 x. The 0.1V curve and the 1 V curve are somewhat less than 3 decades apart, but agree with the pattern, approximately.

A graph was plotted showing a family of asymptotes that meet the criteria, and the asymptotes as a set were moved around to make a "best fit" to the hypothetical data. The match is not too bad for the lower voltages, but the hypothetical data is so uncertain that it would be unwise to draw too many conclusions until there is confirming "real" data from bench tests.



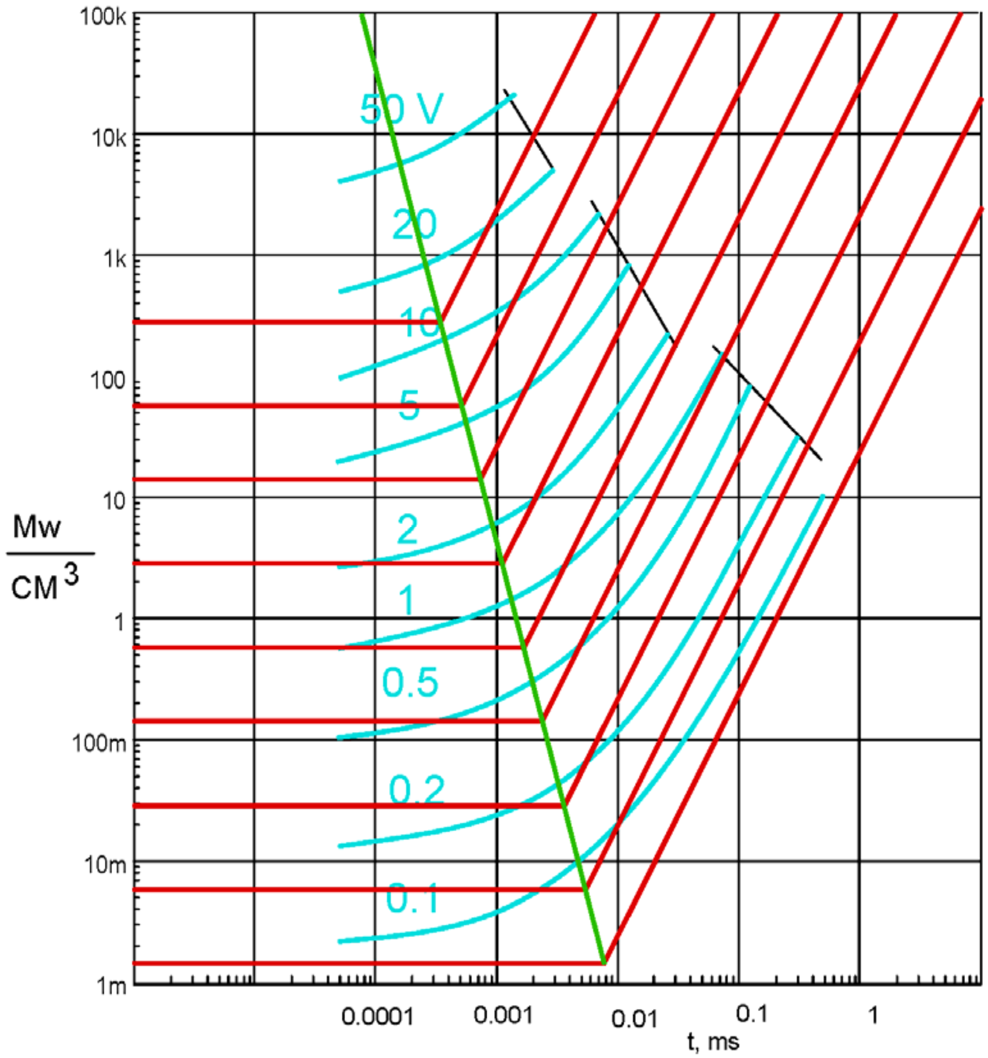
Frequency is not a useful parameter in SPICE, so it is desirable to find the losses as a function of time, that is, in the time domain. Frequency is the inverse of time, the period of an ac cycle. For calculating $\hat{B}$, one half the period is used, so $t = 1/2F$, and $F = 2/t$. Substituting into the equation for $\hat{B}$ gives:

$$\hat{B} = \frac{E*t}{8}*10^{-2} \text{ kG}$$

where t is in milliseconds, to be consistent with the kHz used in the Magnetics, Inc. approximations. Substituting into the approximation gives:

$$Pl = a*(\frac{1}{2*t})^{c}*(\frac{E*t}{8})^{d}$$

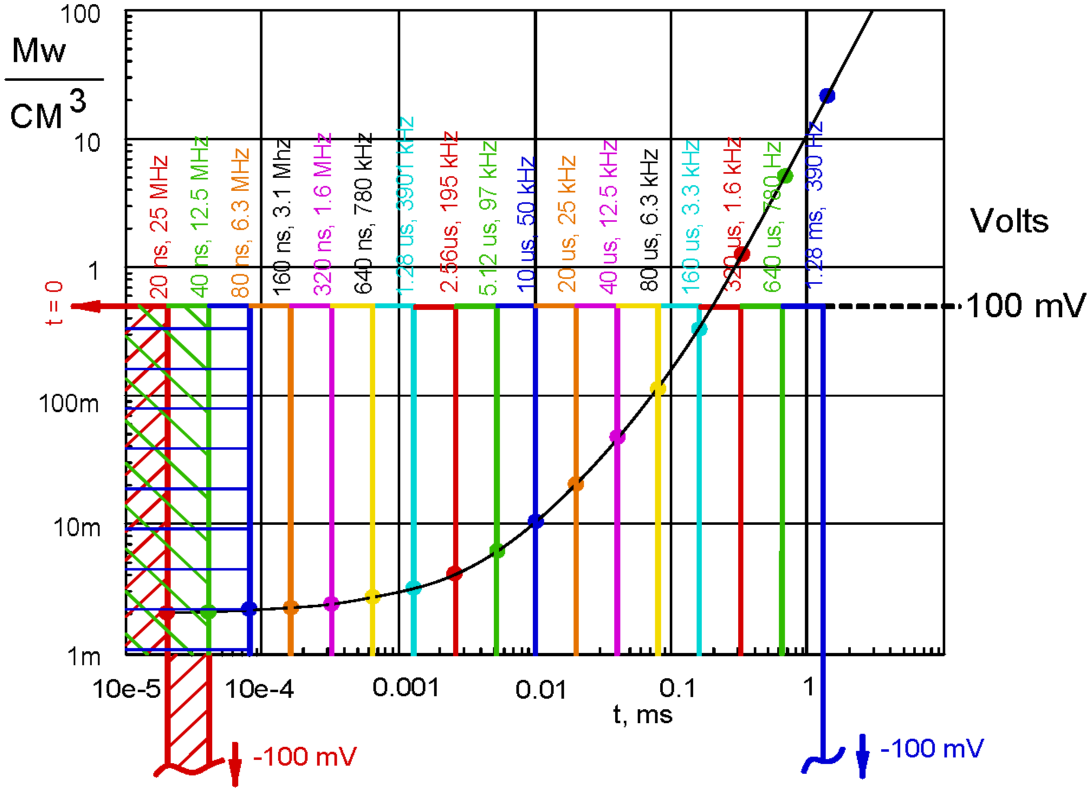
This transformation is easy, graphically, with a left-right mirror image of the curve for loss *vs.* frequency. Match the 1 kHz line to the 500 ms line, so the time is the on time for a half-cycle of the ac frequency.



Assuming that no math errors were made, this graph should predict losses in a waveform expressed in terms of on-time just as well as the graph expressed in frequency. Because the original equation is in terms of $\hat{B}$ (that is, *Bmax* at the end of the on-time), not *B*, instantaneous, this does NOT give us the instantaneous power *vs.* time.

This curve makes a lot more sense as a presentation of core loss data for transformer design, particularly pwm transformers. It is my theory that this graph is also useful for duty-cycles less than 1. In the graph above, find the core loss for the actual voltage and pulse width, then multiply by the duty-cycle. At this time, this is speculation, but if it is confirmed, it will be a powerful new tool for pwm transformer and inductor design.

The curve below builds on the concept of the graph above of power density *vs.* the on-time voltage pulse. Each of the colored curves is a first half of a 100 mV ac square-wave, with the positive going on-time shown, measured from t = 0, part of a continuous ac square wave. In a log-log plot, 0 is infinitely far to the left, so all pulses are drawn from the 10 ns line. The second half of each cycle is symmetrical negative going, part of a continuous ac square wave, but this is shown only for the first pulse and the cross-over of the last pulse, to reduce clutter.



The first pulse, the red line and red hatch, starts at t = 0 at a voltage of 100 mV, and ends at 20 ns, continuing negative to – 100 mv, then starting the next cycle at 40 ns. Thus it defines a square wave having a period of 40 ns or a frequency of 25 MHz. The average power for a 25 MHz ac square wave is placed at the midpoint of the cycle, which is the trailing edge of the positive pulse, and has the same color. The second pulse, the green line and green hatch, starts at t = 0 and ends at 40 ns, so its period is 80 ns and its frequency is 12.5 MHz. The average power for a 12.5 MHz square wave is placed at the center of its cycle, the trailing edge of the positive pulse. The final pulse, a blue line, starts at t = 0 and continues to t = 1.28 ms, for a period of 2.56 ms and a frequency of 390 Hz. The average power for that point is estimated and plotted.

This curve is actually a crude piece-wise integration, and the true instantaneous power curve in the time domain may be the derivative of this curve. This would be a fortunate result for the prospects of making a good SPICE model. Differentiating the curve is daunting, but differentiating the asymptotes is easier.

For low frequencies, the curve of the asymptote is

A $P_L = a * f^1 * \hat{B}^3$ mW/cm^3

For high frequencies, the curve of the asymptote is

B $P_L = a * f^2 * \hat{B}^2$ mW/cm^3

Substituting $f = \frac{1}{2*t}$ and $\hat{B} = \frac{E*t}{8} * 10^{-2}$ kG

Gives:

For low frequencies, the asymptote is:

A $P_L = a * \frac{1}{2*t} * (\frac{E*t}{8} * 10^{-2})^3$ mW/cm^3

A $P_L = a * \frac{1}{2*t} * \frac{E^3 * t^3}{8^3} * 10^{-6}$ mW/cm^3

A $P_L = \frac{a}{512} * E^3 * t^2 * 10^{-6}$ mW/cm^3

Differentiating:

A $P_L' = \frac{a}{256} * E^3 * t * 10^{-6}$ mW/cm^3

To get rid of one of the *E*s in the E^3 and the *t*,

$$B = \frac{E*t}{8} * 10^{-2}$$

$$E*t = 8*B*10^2$$

A $P_L' = \frac{a}{256} * E^2 * (E*t) * 10^{-6}$ mW/cm^3

A $P_L' = \frac{a}{256} * E^2 * (8*B*10^2) * 10^{-6}$ mW/cm^3

A $P_L' = \frac{a}{32} * E^2 * B * 10^{-4}$ mW/cm^3

A $$P_L' = \frac{E^2}{R_B} \qquad \text{mW/cm}^3$$

where $R_B = \dfrac{32}{a*B}$ for $B > 0$.

This can be modeled in SPICE as the voltage across a resistor. The value of the resistor R_B is determined by a behavioral source as a function of the inverse flux or flux density *B*. However, this is not the same flux or flux density *B* used in the saturation model. It will have to be modeled separately, as it resets for each voltage reversal.

For high frequencies, the asymptote is:

B $$P_L = a * f^2 * \hat{B}^2 \qquad \text{mW/cm}^3$$

Substituting $f = \dfrac{1}{2*t}$ and $\hat{B} = \dfrac{E*t}{8} * 10^{-2}$ kG

B $$P_L = a * (\frac{1}{2*t})^2 * (\frac{E*t}{8})^2 \qquad \text{mW/cm}^3$$

The t^2s cancel, leaving

B $$P_L = \frac{a}{256} * E^2 \text{ mW/cm}^3$$

That is a constant, so the derivative is the same.

B $$P_L' = \frac{E^2}{R_h} \qquad \text{mW/cm}^3$$

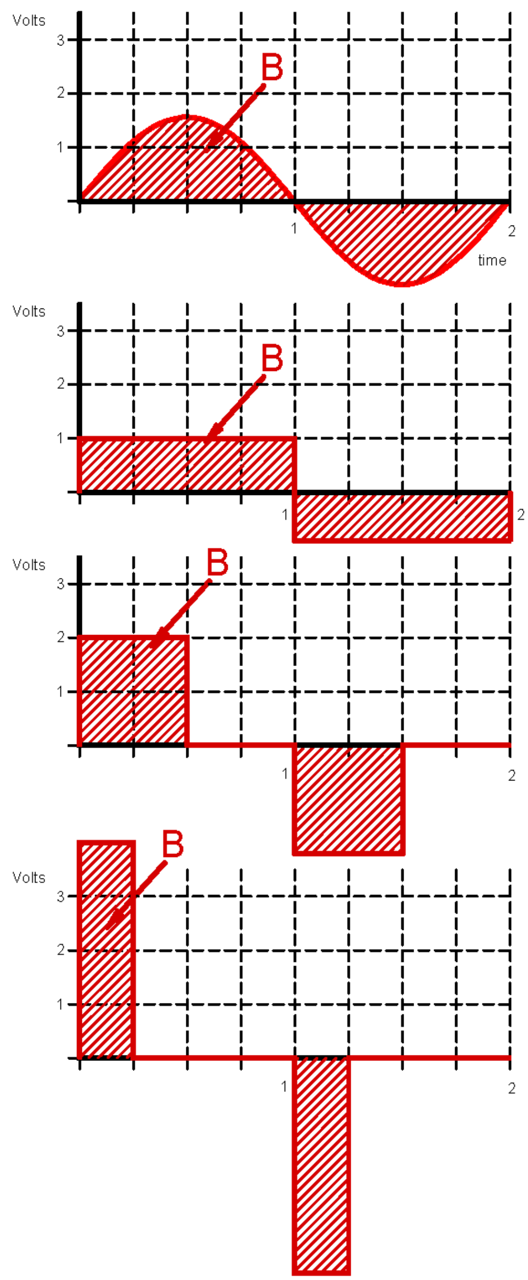
where $R_h = 256/a$,

Thus the high frequency SPICE model is simply the voltage across a resistor.

That is a lot of equations with many chances to make errors. If correct, then the core loss function can be modeled as two resistors modeling the asymptotes for the low and high frequency cases. The resistor that dominates at low frequency is a function of 1/*B*.

A real core has an inductance which is complex. The real part is purely inductive and the imaginary part is resistive, that is, it is lossy. Both components have significant non-linearity, so their counterparts in a SPICE model may need corrective curve-fitting functions or step-wise approximations. It may not be possible to make a SPICE model that is accurate over very much of a range of operation.

Low frequency core losses, graphical solution:



(This analysis was done first, and provided the insight for parsing the manufacturers' data.)

Knowing the strong relationship between $\hat{B}$ and the core loss, it is intuitively reasonable that the core loss would be much higher at low frequencies with a constant voltage square-wave applied. However, it is not at all intuitive, to me at least, what impedance model, if any, would produce that result.

The graphs at the right show four wave-shapes having the same period and having the same $\hat{B}$, therefore all would have the same core loss at low frequencies. Note: these graphs show $\hat{B}$, the maximum flux density, not *B*, the instantaneous flux density.

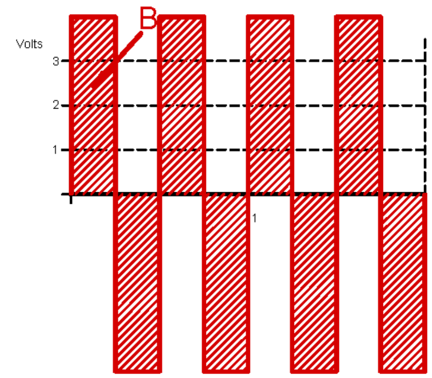
It seems reasonable that core losses would occur only when there is voltage applied to the winding. In the second graph, 1 volt is applied for 1 unit of time. The core loss presumable occurs over the entire pulse width, say 1 watt.

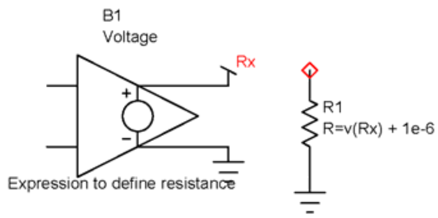
In the third graph, 2 volts are applied for half the time. Since the average loss is the same, the loss during the pulse must be 2 watts. Similarly in the fourth graph, 4 volts are applied for one fourth the time, and the power must be 4 watts to make the average correct.

Thus, the core loss is directly proportional to the voltage, V/Y, where Y is a yet to be defined resistance function. With a fixed resistor, the power relationship is the voltage squared, V^2/R.

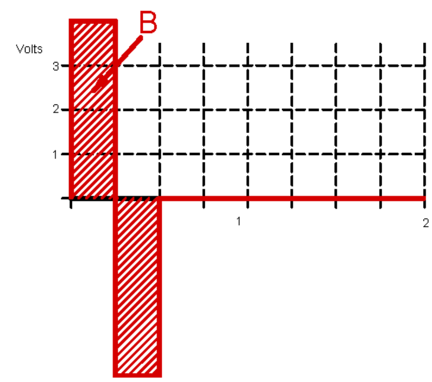
Power is the product of current and voltage. With a resistor, it can also be expressed as V^2/R or I^2R. With a uniform voltage, a varying power could be modeled using a variable current source or a variable resistor. If the instantaneous power is uniform with time, a resistor X that varied as the voltage would produce the power suggested above, V^2/X, if $X = V*R_1$, where R_1 is the value at V = 1 volt.

We still do not know if the losses are uniform over the time that voltage is applied, and other data suggests that it is not. Whatever the curve, though, to produce the result observed at low frequency, it must scale with voltage and compress with time uniformly across the examples.

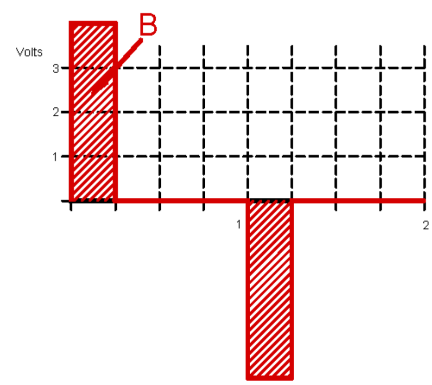




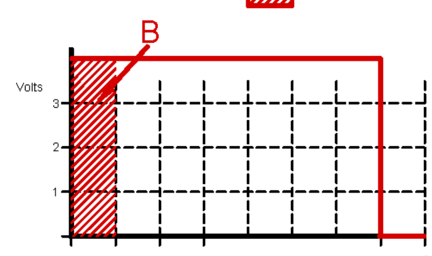
In SPICE models, the values of resistors (and other components) can be variable, defined by expressions. Rather than put a complicated expression in the resistance definition, I prefer to generate a voltage in a behavioral voltage source, and set the resistance by entering R=v(Rx), where Rx is the voltage of the behavioral source.



This allows different parameters to be brought into the expression for the value of the resistance, which can be useful for curve fitting. The power in the resistor is calculated as the current times the voltage and the product can be summed with the power of other resistors in the core loss model to give the total core loss as a test point. (In a similar manner, the value of inductors and capacitors can be controlled by expressions, but that is not recommended if the energy stored in the component must be conserved.)

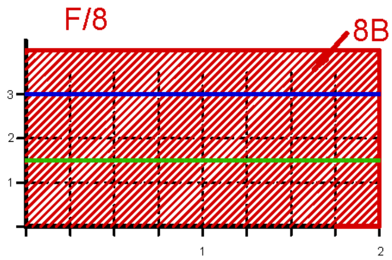


If pulses are viewed in isolation, are the losses the same? Or are there some history phenomena? At the right are five graphs of different waveforms. For the same voltage and time (volt-seconds, or B), are the losses always the same? What happens with losses at the beginning of a pulse? The voltage is applied, and the core starts to dissipate power, but at what level? The duration of the pulse is not yet known, so it could be 500 ns (1 MHz) or 500 ms (1 Hz), the initial conditions are the same. It seems that the losses in the initial 500 ns must be those consistent with a 1 MHz frequency, that is, quite low. After 5 us, the power must be at a level consistent with a 100 kHz frequency, and after 50 us, the power must be at a level consistent with a 10 kHz frequency. That is, after voltage is applied, the losses must be quite low at first and rise as time goes on.



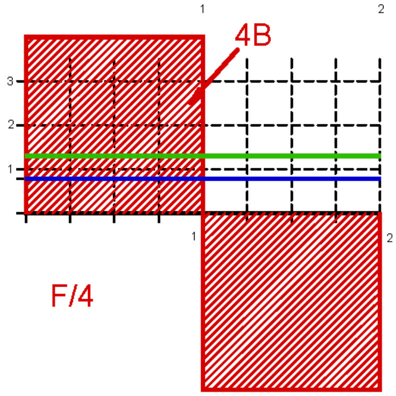
The graphs below show four waveforms. With a constant voltage applied, they have an binary relationship, *8xB, 4xB, 2xB* and *B*, also *F/8, F/4, F/2* and *F*. The green and blue lines show the relative power if the graphs represent low frequency excitation (blue) and high frequency excitation (green). Note: These graphs show $\hat{B}$, the maximum flux density, not *B*, the instantaneous flux density.

<u>Frequency, excitation, power density</u>



1 kHz, 100 mV, 10 mW/cm^3.

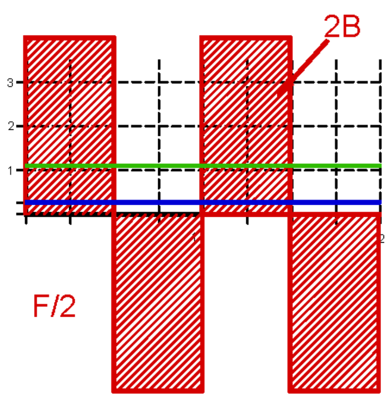
1 MHz, 500 mV, 0.17 mW/cm^3.



2 MHz, 500 mV, 0.15 mW/cm^3.

2 kHz, 100 mV, 3.0 mW/cm^3.

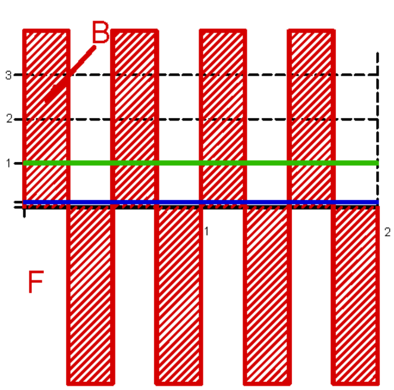
4 MHz, 500 mV, 0.13 mW/cm^3.



4 kHz, 100 mV, 0.8 mW/cm^3.

8 MHz, 500 mV, 0.12 mW/cm^3.

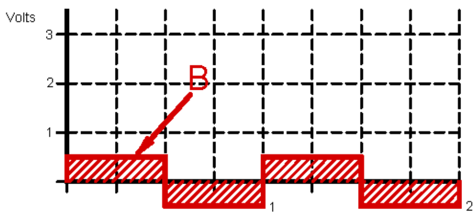
8 kHz, 100 mV, 0.33 mW/cm^3.

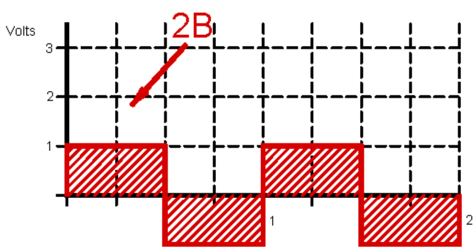


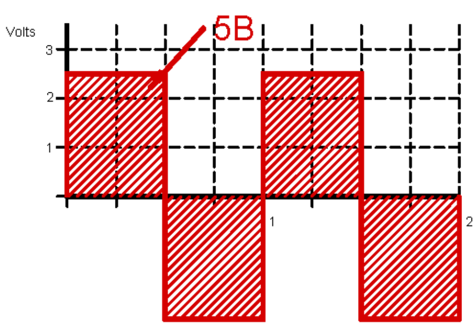
The values are from the graph of loss *vs.* frequency for various applied voltages, above. Note that the average power does not change very much at high frequency, but changes dramatically at low frequency for the same relative changes in B and F.

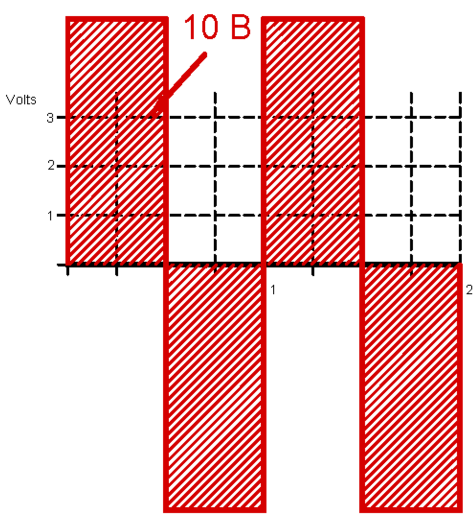
The graphs below show the voltage and time for maximum flux densities of B, 2xB, 5xB and 10xB at 10 kHz and 10 MHz. The data is very uncertain at 10 MHz, but is included for qualitative purposes. For each condition, the voltage and the power is given, both on the curve and for the asymptote (with reference to the graph above of loss density . frequency.

For the lower frequency, 10 kHz, a 10 times increase in B results in about a 1000 times increase in the loss power density, or three orders of magnitude. At 10 MHz, a 10 times increase in B results in about a 100 times increase in the loss power density.









Note: These graphs show $\hat{B}$, the maximum flux density, not *B*, the instantaneous flux density.

Frequency Hz	Voltage V	Curve mw/cm^3	Asymptote mw/cm^3
10 M	100 m	.0.0024	0.0014
10 k	100 m,	0.018	0.07
10 M	200 m	0.014	0.006
10 k	200 m	1.1	0.5
10 M	500 m	0.11	0.03
10 k	500 m	13.0	4.0
10 M	1.0	0.18	0.14
10 k	1.0	80	70

Returning to the consideration of the low frequency power loss relationships, with fixed $\hat{B}$ and variable frequency, the power density varied as V/X, where X is an undefined resistance function. By contrast, the usual voltage relationship to power is V^2/R.

With fixed frequency and a variable $\hat{B}$, the power density varies as $\hat{B}^3$, while the voltage varied as V. Since $\hat{B}$ is volt-seconds, and the time is constant, then the power was varying as V^3. If the power varies as V with $\hat{B}$ fixed and as V^3 with $\hat{B}$ varying, it seems that V^2 is attributable to the varying $\hat{B}$. Thus it seems that assuming that the core loss varied as $\hat{B}^3$ is incorrect, it has to be:

$$P_L = \frac{V * \hat{B}^2}{X}$$

where X is a constant, a scale factor.

To express this as a function of time, so that it can be differentiated, consider that $\hat{B}$ is volt-seconds. Substituting $\hat{B} = V * t$, we get:

$$P_L = \frac{V^3 * t^2}{X}$$

Taking the derivative:

$$P_L' = \frac{2 * V^3 * t}{X}$$

This can be restated as

$$P_L' = \frac{2 * V^2 * (V * t)}{X}$$

$$P_L' = \frac{2 * V^2 * B}{X}$$

Note, having been differentiated, P_L' is instantaneous power density and B is the instantaneous flux density.

This has the form V^2/R if R varies as $1/B$.