Modeling of Piezoelectric and Acoustic Elements with LTspice\SwCad

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The LTspice\SwCad model for an ultrasonic piezoelectric transducer and acoustic medium is described. This model allows the simultaneous evaluation of the electric and acoustic characteristics of a complex piezoelectric system with an acoustic load. This model is very useful for the investigation of the acoustic and electric coupling. It also helps to optimize the transducer's power amplifier for best acoustic effectiveness.

The described model is based on previous SPICE and PSPICE models [1-4], which use a model of the acoustic medium adapted to an electric transmission line. In analogy to the equivalent circuit of a transmission line, the model proposed represents the piezoelectric thickness vibrator by lumped elements, i.e., capacitor, inductor, and current-controlled current source. The simulation of the model is realized by applying standard LTspice electrical network analysis techniques. In order to unify electric and acoustic theory, an impedance type analogy is chosen where mechanical force is represented by voltage and particle velocity is represented by current.

The physical and electro-mechanical data for the calculation of the model parameters can be obtained from any suitable handbook.

1. The Basic Equations

The input data and notation used in the equations.

Acoustic system elements geometry:

l - length (thickness){*m*}; ***S*** - cross-sectional area {m^2}.

Medium mechanical characteristics:

ρ - density {kg/m^3}; ***v*** - sound speed {*m/s*}; ***α*** - acoustic coefficient of attenuation {*Np/m*}.

Piezoelectric electro-mechanical characteristics:

εS - dielectric permittivity with zero strain {C/Nm^2};

e33 - piezoelectric tensor component {C/m^2}.

The electro-acoustic analogous equations for the Lossy Transmission Line - LTRA.

$L = \rho \cdot S$ - line inductance per unit length {*H/m*}.

$C = \frac{1}{v^2 \cdot \rho \cdot S} = \frac{1}{v^2 \cdot L}$ - shunt capacitance per unit length {*F/m*}.

$z = \sqrt{\frac{L}{C}} = v \cdot \rho \cdot S$ - wave resistance {*Ω*}. This resistance corresponds to acoustic (mechanical) impedance of infinite medium.

$f_0 = \frac{v}{2 \cdot l}$ - resonance frequency {Hz}.

$Z_0 = 2\pi \cdot f_0 \cdot L = \frac{1}{2\pi \cdot f_0 \cdot C}$ - reactance per unit length {*Ω/m*} for resonance frequency.

This is equal to expression - $Z_0 = 2\pi \cdot f_0 \cdot L = 2\pi \cdot \frac{v}{2 \cdot l} \cdot \rho \cdot S = \frac{2\pi \cdot z}{2 \cdot l}$.

$Q = \frac{Z_0}{R}$ - piezoelectric quality (Q-factor) for resonator.

There are two forms suitable for line resistance per unit length *R* {*Ω/m*} representation:

$R = 2 \cdot v \cdot \rho \cdot S \cdot \alpha$ - for an ordinary acoustic medium.

$R = \frac{Z_0}{Q} = \frac{2\pi \cdot f_0 \cdot L}{Q}$ - for a piezoelectric resonator.

For most viscous fluids (water for example) the attenuation can be calculated by the expression – $\alpha = \dfrac{2\cdot(2\pi\cdot f)^2}{3\cdot\rho\cdot v^3}\cdot\eta$.

Here η- viscosity coefficient; f- working frequency.

In addition we need some relations for piezoelectric resonator by Leach [3] model:

$C_0 = \dfrac{\varepsilon S\cdot S}{l}$ - shunt piezoelectric resonator capacity {F}. This is common static capacity of piezoelectric resonator.

$h1 = \dfrac{e33}{\varepsilon S}$ - electric current and strain relation in piezoelectric.

$h2 = h1\cdot C_0$ - voltage and current relation in piezoelectric.

There are three well known but different forms of acoustic impedance:

Specific acoustic impedance - $Z_1 = \dfrac{T}{u}$.

Here are: T - tension or pressure; u – particles velocity.

Acoustic impedance - $Z_a = \dfrac{Z_1}{S}$. $Z_a = \dfrac{T}{u\cdot S}$.

This form is used for the description of acoustic vibrations in tubes with variable cross sectional area. $u\cdot S = \dfrac{dV}{dt}$ - volume vibration speed, here are: S- cross-sectional area, V-deformation volume.

Mechanical impedance $Z_m = Z_1\cdot S = Z_a\cdot S^2$. $Z_m = \dfrac{T\cdot S}{u} = \dfrac{F}{u}$.

Here is F-mechanical force.

The model described uses the mechanical form of acoustic impedance. Fig.1 demonstrates the main relations between the forces F_{ij} and deformations ξ_i ($u = \dfrac{d\xi}{dt}$) in case of three interacting elements 1, 2, 3, with different cross-sectional areas S_i.

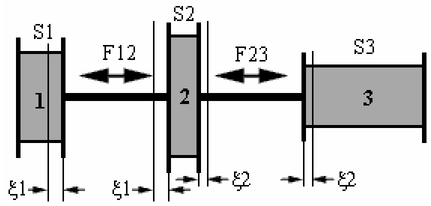
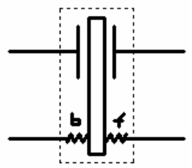
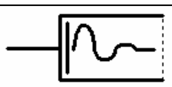


Fig.1. The acoustic elements interacting.

Based on these relations we have defined three basic elements as shown below.

	***PiezoElement.asy*** – model of piezoelectric resonator with electric and acoustic (b, f) terminals.
	***SubstFin.asy*** – model of finite acoustic medium with certain absorption and resonance properties.
	***SubstInf.asy*** – model of infinite acoustic medium with certain acoustic impedance.

2. The Example Schematics

The Electro-acoustic Echo System.

Here is an example of a schematic with electric and acoustic paths.

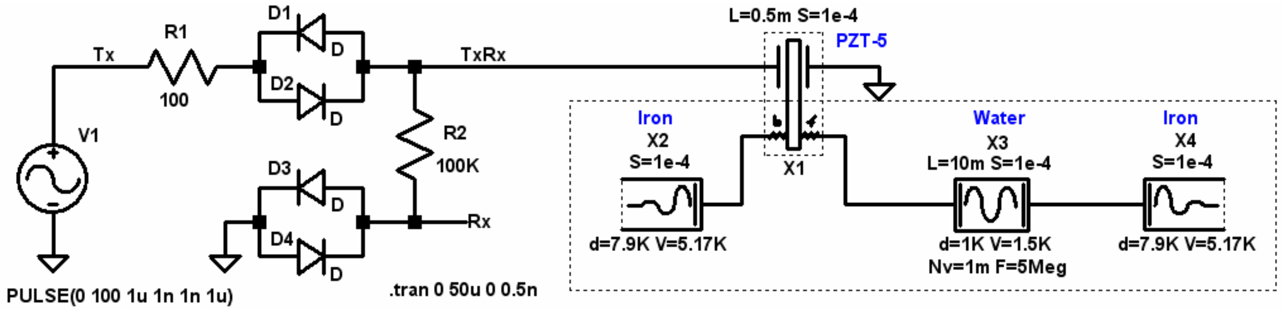


Fig.2. Test schematic for pulse echo observation.

The left-hand side of the schematic consists of the pulse generator and diode limiters. The right-hand side of the schematic represents the acoustic path with the piezoelectric transducer (PZT-5A material). The acoustic path consist of two infinite iron layers X2, X4; transducer body X1, with thickness L=0.5mm; and water layer X3 L=10mm. All acoustic elements cross-sectional area is 1cm^2 (S=1e-4m^2). The *V* parameter under X2, X3, X4 is sound speed and *d* is density (Water: V=1500m/s, d=1000kg/m^3; Iron: V=5170m/s, d=7900kg/m^3).

Fig.3 shows the voltage of echo pulses in Tx, TxRx and Rx of the electrical part in the top graphs. The mechanical deformation is shown in the bottom graph– speed of particles (current) in water near the iron.

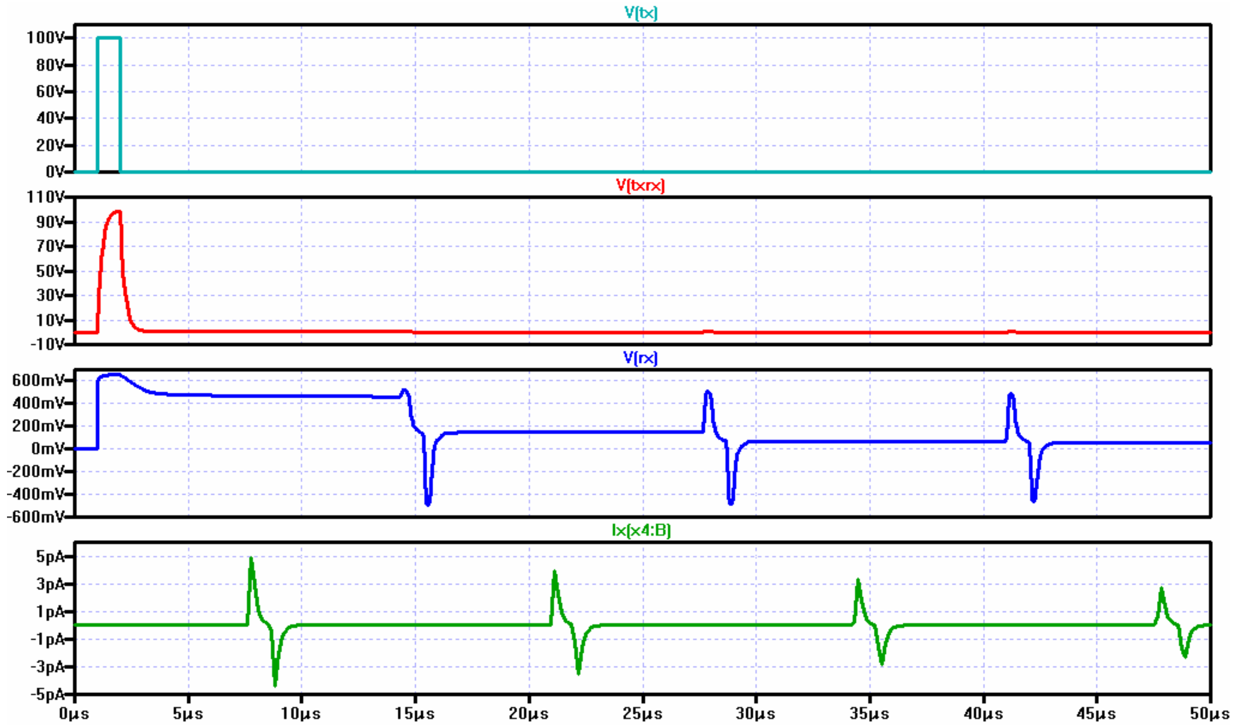


Fig.3. Echo pulses in test schematic.

The Piezoelectric Resonator.

This is an example of a schematic for a free Piezoelectric Resonator with the Air round-up.

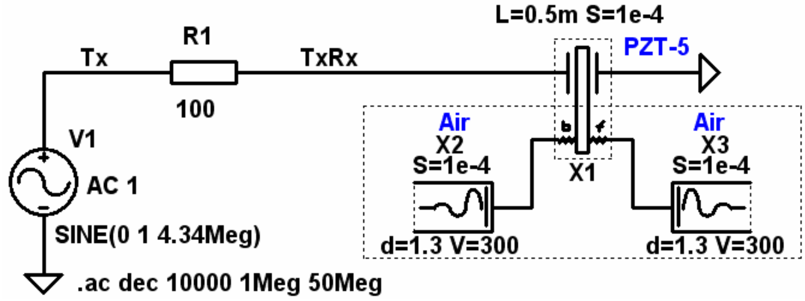


Fig.4. Test schematic for free Piezoelectric Resonator with the Air round-up.

Fig.5 represents the frequency dependence of voltage and current on the Piezoelectric Resonator (top pictures), and acoustic deformation velocity (deformation current) on the Piezoelectric Resonator surface (bottom picture).

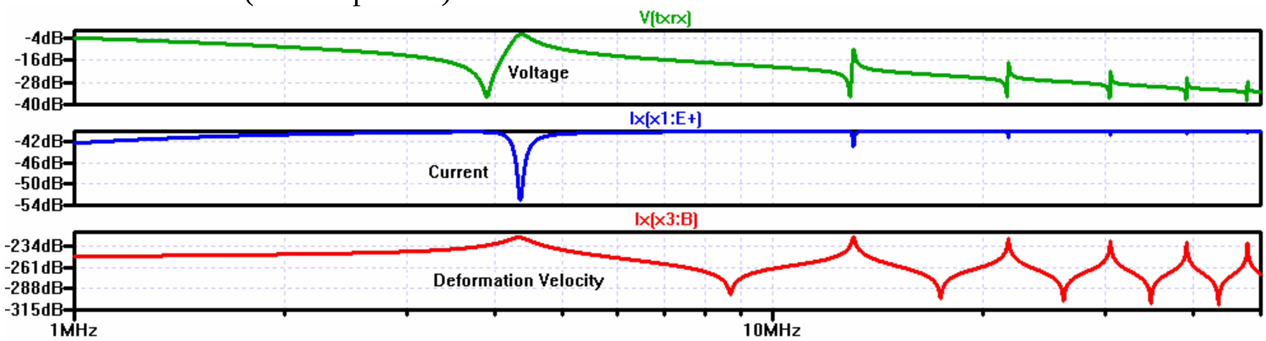


Fig.5. Frequency dependence of Voltage, Current and Deformation Velocity of a free Piezoelectric Resonator with the Air round-up.

3. The Piezoelectric and Acoustic Elements Models

PiezoElemen model

The LTspice-model of the Piezoelectric transducer is a slightly modified adaptation of the SPICE-model, described by J.Deventer [4]. This model consists of: LTRA-Lossy Transmission Line; current and voltage dependent sources; several additional elements.

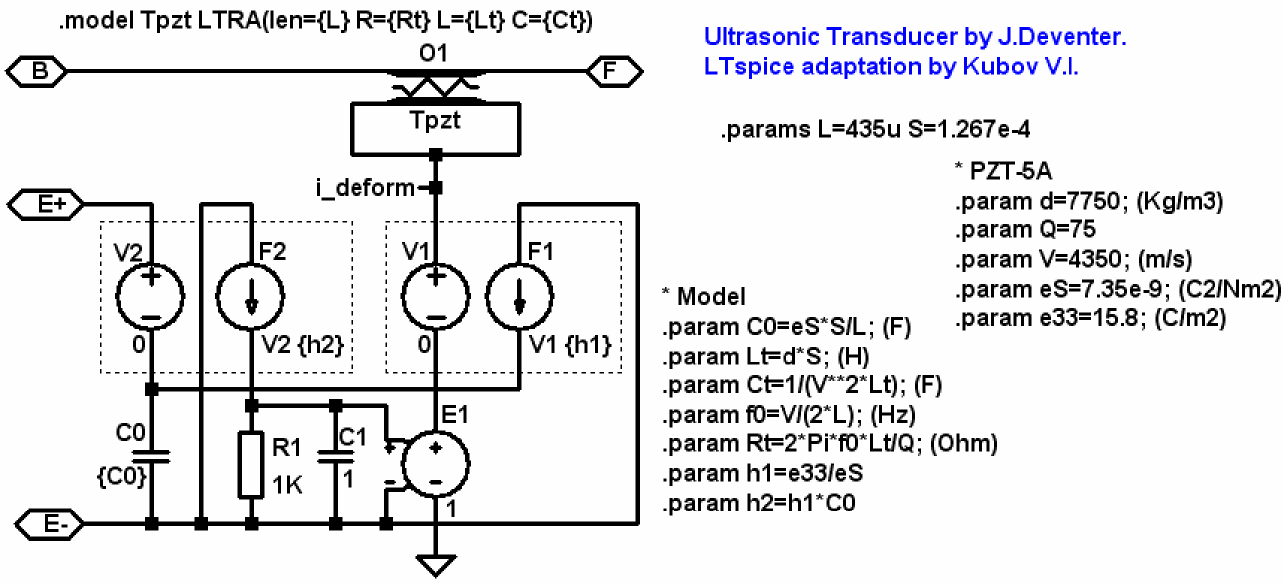


Fig.6. PiezoElement.asc model schematic.

The model has electric path terminals “E+”, “E-“ and acoustic path terminals “b”-back, “f”-forward. It consists of three parameter sets:

Model formulas	Material data
.model Tpzt LTRA(len={L} R={Rt} L={Lt} C={Ct}) * Model .param C0=eS*S/L; (F) .param Lt=d*S; (H/m) .param Ct=1/(V**2*Lt); (F/m) .param f0=V/(2*L); (Hz) .param Rt=2*Pi*f0*Lt/Q; (Ohm/m) .param h1=e33/eS .param h2=h1*C0	* PZT-5A .param d=7750; (Kg/m3) .param Q=75 .param V=4350; (m/s) .param eS=7.35e-9; (C2/Nm2) .param e33=15.8; (C/m2) **Transducer geometry** .params L=435u S=1.267e-4

Any material data or transducer geometry can be changed on the main schematic as elements' parameters.

SubstFin model

The Model of a finite acoustic medium with certain absorption and resonance properties consists of the single LTRA-Lossy Transmission Line. This model includes formulas for absorption calculations with frequency dependence for most viscous fluids such like water.

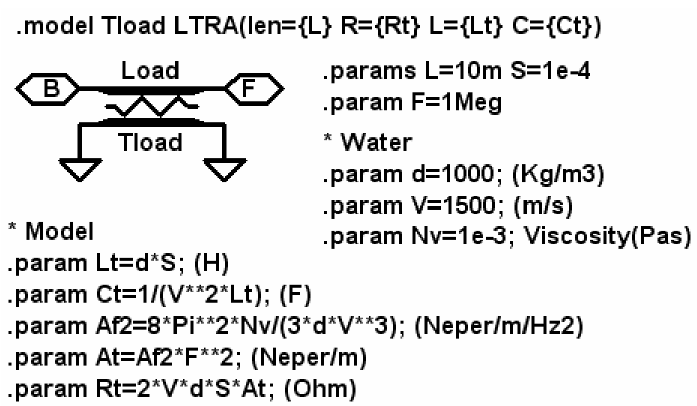


Fig.7. SubstFin.asc model schematic.

The Model has only acoustic path terminals “b”-back, “f”-forward.
It consists of four parameter sets:

Model formulas	Material data
.model Tload LTRA(len={L} R={Rt} L={Lt} C={Ct}) * Model .param Lt=d*S; (H/m) .param Ct=1/(V**2*Lt); (F/m) .param Af2=8*Pi**2*Nv/(3*d*V**3); (Neper/m/Hz2) .param At=Af2*F**2; (Neper/m) .param Rt=2*V*d*S*At; (Ohm/m)	* Water .param d=1000; (Kg/m3) .param V=1500; (m/s) .param Nv=1e-3; Viscosity(Pas) **Medium geometry** .params L=10m S=1e-4 **Work frequency** .param F=1Meg

The absorption parameters in the formulas can be set directly with values from the data book. This is suitable in case of unknown viscosity data or absorption frequency dependence.

SubstInf model

The model of an infinite acoustic medium consists of a single resistor with calculated impedance.

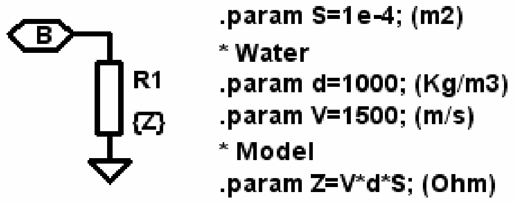


Fig.8. SubstInf.asc model schematic.

This Model has single acoustic path terminals “b”-back.
It consists of three parameter sets:

Model formulas	Material data
* Model .param Z=V*d*S; (Ohm)	* Water .param d=1000; (Kg/m3) .param V=1500; (m/s)
	Medium geometry
	.params S=1e-4; (m2)

4. References

1. R. Schwarz. "Digital computer simulation of a piezoelectric thickness vibrator". J. Acoustical Soc. Am., vol.62, No 2, 1977, pp.463-467.
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3. W. M. Leach, Jr. "Controlled-source analogous circuits and SPICE models for piezoelectric transducers". IEEE Trans. Ultrason. Ferroelect. Freq. Contr., 1984, v. 41, 60–66.
4. C.J van Deventer. Doctoral thesis. “Material Investigations and Simulation Tools Towards a Design Strategy for an Ultrasonic Densitometer”. Lulea University of Technology, 2001. “Modeling an Ultrasonic Transducer with SPICE”. Paper C. http://epubl.luth.se/1402-1544/2001/31/LTU-DT-0131-SE.pdf

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